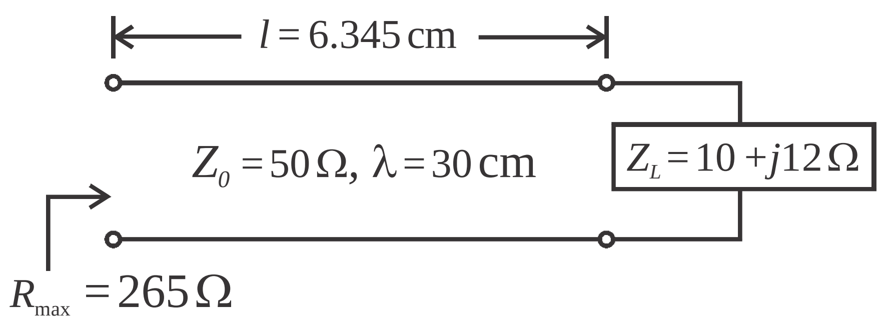
Match a $10 + j\,12\,\Omega$ load to a $50\,\Omega$ microstrip transmission line (λ=30 cm) using a quarter-wave transformer and $50\,\Omega$ microstrip. Restriction- the match should be as short as possible.

1) Normalize Z_L and plot on Smith chart

- Normalize $z_L = Z_L / Z_0 = (10 + j\,12) / 50 \;\Rightarrow\;$ **$z_L = 0.2 + j\,0.24\ \Omega/\Omega$**.
- Plot z_L on Smith chart by finding intersection of $r = 0.2$ circle & $x = 0.24$ arc.

2) Find first point along $50\,\Omega$ microstrip where the impedance is real

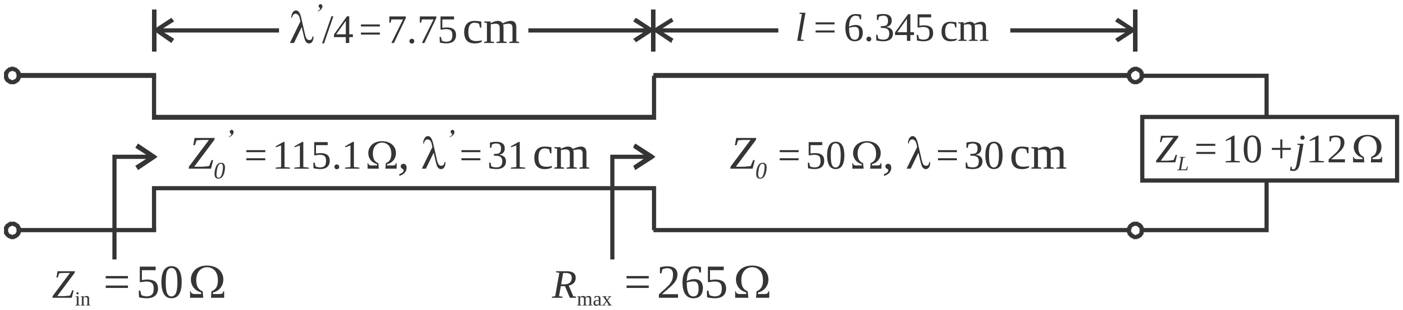
- Use compass to draw arc of constant $|\Gamma|$ from z_L point on Smith chart in the "WAVELENGTHS TOWARD GENERATOR" direction until reaching the horizontal/real axis to right of origin.
- Read $r_{\max} = 5.3$ on Smith chart. This corresponds to $R_{\max} = R_{\max}\,Z_0 = (5.3)\,50$ $\Rightarrow$ **$R_{\max} = 265\ \Omega$**.
- Find distance from z_L to $r_{\max}$ by drawing radial line from the center of Smith chart through z_L and the "WAVELENGTHS TOWARD GENERATOR" scale, reading 0.0385 and noting $r_{\max}$ is at 0.25 on the scale. The distance $l = (0.25\text{-}0.0385)\lambda = 0.2115\lambda \Rightarrow$ **$l = 6.345$ cm**.

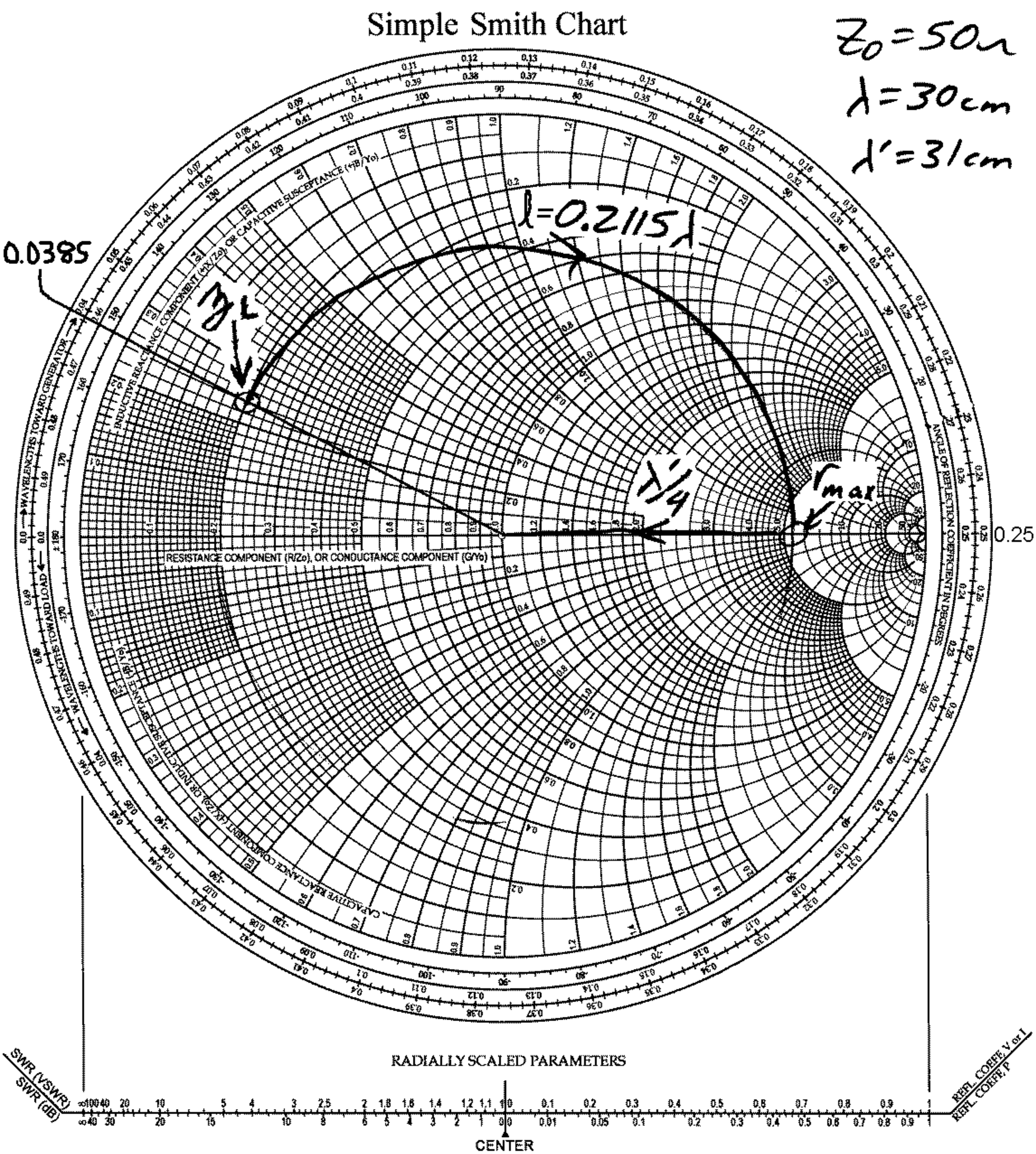


- Now that we have a real impedance, we can use a quarter-wave transformer to match to $50\,\Omega$ (next step).

3) Design quarter-wave transformer to match $R_{\max}$ to $50\,\Omega$

- Use equation (11.58) of text to find characteristic impedance of quarter-wave transformer $Z_0' = \sqrt{Z_0 Z_L} = \sqrt{Z_0 R_{\max}} = \sqrt{50(265)} \Rightarrow$ **$Z_0' = 115.109\ \Omega$**.
- By definition, a quarter-wave transformer will have a length of $\lambda'/4$. The wavelength λ' on 115.1 Ω microstrip will NOT be the same as λ=30 cm on 50 Ω microstrip (Note: wavelength for microstrip depends on circuit board material and thickness as well as microstrip width, see section 11.8 of text). For the sake of this example, assume λ'=31 cm. Hence, **$\lambda'/4$=7.75 cm**.


λ'/4 = 7.75 cm
l = 6.345 cm
Z0' = 115.1 Ω, λ' = 31 cm
Z0 = 50 Ω, λ = 30 cm
ZL = 10 + j12 Ω
Zin = 50 Ω
Rmax = 265 Ω


Simple Smith Chart
Z0=50Ω
λ=30cm
λ'=31cm
0.0385
yL
l=0.2115λ
λ'/4
rmax
0.25
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
RADIALLY SCALED PARAMETERS
CENTER