λ/4 Monopole Antenna for AM radio example

Assume the monopole operates at 920 kHz and is driven with a current of 100 A. First, find the time-average Poynting vector. Then, find the power density and how much power would flow through a 1 m by 1 m window at ground level 5 km from antenna.

$$\lambda = u/f = \frac{2.998\times10^8}{920\times10^3} = 325.87\ m$$

$$\beta = \frac{2\pi}{\lambda} = 0.01928\ \frac{rad}{m}$$

Using (13.19)

$$\bar{H} = \hat{a}_\phi \frac{j I_0 e^{-j\beta r}\cos(\pi/2\cos\theta)}{2\pi r\sin\theta} = \hat{a}_\phi \frac{j100e^{-j0.01928r}\cos(\pi/2\cos\theta)}{2\pi r\sin\theta}$$

$$= \hat{a}_\phi \frac{j15.9155\, e^{-j0.01928r}\cos(\pi/2\cos\theta)}{r\sin\theta}\ \left(A/m\right)$$

$$\bar{E} = \eta(\bar{H}\times\hat{a}_k) = \eta_0(\hat{a}_\phi H_\phi \times \hat{a}_r) = \hat{a}_\theta \eta_0 H_\phi$$

$$= \hat{a}_\theta\, 376.73\left(\frac{j15.9155e^{-j0.01928r}\cos(\pi/2\cos\theta)}{r\sin\theta}\right)$$

$$= \hat{a}_\theta \frac{j\,5995.85\, e^{-j0.01928r}\cos(\pi/2\cos\theta)}{r\sin\theta}\ \left(V/m\right)$$

$$\bar{P}_{ave} = \tfrac{1}{2}\,\mathrm{Re}\{\bar{E} \times \bar{H}^*\}$$

$$= \tfrac{1}{2}\,\mathrm{Re}\left\{\hat{a}_\theta \frac{j5995.85\, e^{-j0.01928r}\cos(\frac{\pi}{2}\cos\theta)}{r\sin\theta} \times \hat{a}_\phi \frac{-j15.9155\, e^{+j0.01928r}\cos(\frac{\pi}{2}\cos\theta)}{r\sin\theta}\right\}$$

$$= \tfrac{1}{2}\,\mathrm{Re}\left\{\hat{a}_r \frac{-(-1)\,95{,}426.968\, e^{0}\cos^2(\frac{\pi}{2}\cos\theta)}{r^2\sin^2\theta}\right\}$$

$$\bar{P}_{ave} = \hat{a}_r \frac{47{,}713.48\cos^2(\frac{\pi}{2}\cos\theta)}{r^2\sin^2\theta} \quad \left(\frac{W}{m^2}\right)$$

On ground $\Rightarrow \theta = \pi/2 = 90°$ at $r = 5000\,m$

$$\bar{P}_{ave} = \hat{a}_r \frac{47713.48\cos^2(0)}{5000^2\sin^2(\pi/2)} = \hat{a}_r\, 1.90854\ \frac{mW}{m^2}$$

$|\bar{P}_{ave}|$ = power density $= 1.909\ \frac{mW}{m^2}$

$$\text{Power} = \int_{S_{window}} |\bar{P}_{ave}|\, ds = |P_{ave}|\,(\text{Area}) = 1.909\ \frac{mW}{m^2}\,(1\,m^2)$$

$$\text{Power} = 1.909\,mW$$