Polarization Examples

- Arbitrarily choose the z = 0 plane to plot the polarization ellipse as this makes the '$-\beta z$' term zero rather than some other constant.
- Arbitrarily choose to select axes for the plot of the polarization ellipse such that the wave is propagating into the page.
- Plot the polarization ellipse over one period (T) of time, i.e., let 'ωt' term go from 0 to 2π.

Linear Polarization

Assume we have a UPW where the electric field is given by

$$\bar{\mathcal{E}}(z,t) = \hat{a}_x 50\cos(\omega t - \beta z) - \hat{a}_y 60\cos(\omega t - \beta z) \ \text{(V/m)}.$$

From the '$-\beta z$' term, we know the UPW is propagating in the +z-direction. On the z = 0 plane, the equation for the UPW simplifies to

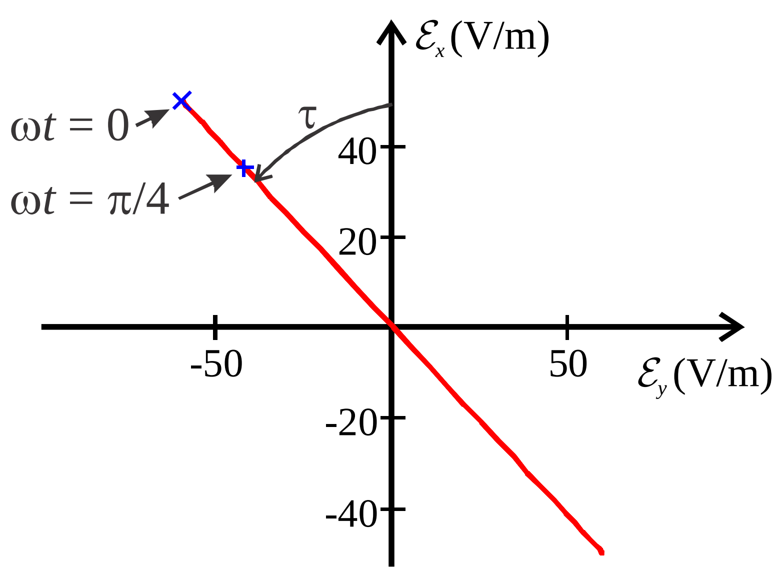
$$\bar{\mathcal{E}}(0,t) = \hat{a}_x 50\cos(\omega t) - \hat{a}_y 60\cos(\omega t) \ \text{(V/m)}.$$

So, the electric field has two orthogonal components that are unequal in magnitude and ±π (±180°) out of phase [trig identity -cos(A) = cos(A ± π)].

Use MathCad & CorelDraw to plot the polarization ellipse with $\mathcal{E}_x$ as the vertical axis and $\mathcal{E}_y$ as the horizontal given so that the +z-direction ($\hat{a}_x \times \hat{a}_y = \hat{a}_z$) is into the page.

$$n := 0..72 \quad \omega t_n := \frac{2\cdot\pi\cdot n}{72} \quad Ex\_lin_n := 50\cdot\cos(\omega t_n) \quad Ey\_lin_n := -60\cdot\cos(\omega t_n)$$

ωt (rad)	$\mathcal{E}_x$(V/m)	$\mathcal{E}_y$(V/m)
0	50cos(0) = 50	-60cos(0) = -60
π/4	50cos(π/4) = 35.36	-60cos(0) = -42.43



Note: **<u>Sense N/A</u>**, axial ratio **<u>AR = ∞</u>**, & tilt angle wrt x-direction is **<u>τ ≈ 50°</u>**.

Circular Polarization

Assume we have a UPW where the electric field is given by

$$\bar{\mathcal{E}}(z,t) = \hat{a}_x 10\cos(\omega t - \beta z) + \hat{a}_y 10\cos(\omega t - \beta z - \pi/2) \ \text{(V/m)}.$$

From the '$-\beta z$' term, we know the UPW is propagating in the +z-direction. On the z = 0 plane, the equation for the UPW simplifies to

$$\bar{\mathcal{E}}(0,t) = \hat{a}_x 10\cos(\omega t) + \hat{a}_y 10\cos(\omega t - \pi/2) \ \text{(V/m)}.$$

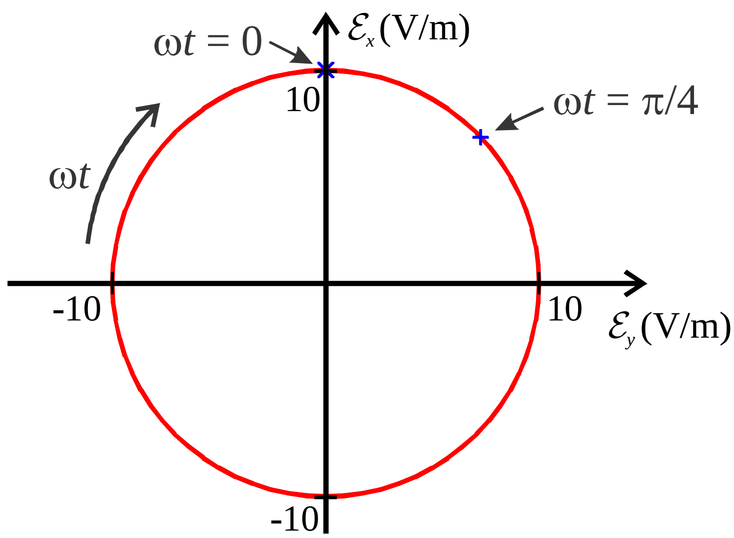
So, the electric field has two orthogonal components that are equal in magnitude and -π/2 (-90°) out of phase.

Use MathCad & CorelDraw to plot the polarization ellipse with $\mathcal{E}_x$ as the vertical axis and $\mathcal{E}_y$ as the horizontal given so that the +z-direction ($\hat{a}_x \times \hat{a}_y = \hat{a}_z$) is into the page.

$$n := 0..72 \qquad \omega t_n := \frac{2\cdot\pi\cdot n}{72}$$

$$Ex\_cir_n := 10\cdot\cos(\omega t_n) \qquad Ey\_cir_n := 10\cdot\cos\left(\omega t_n - \frac{\pi}{2}\right)$$

ωt (rad)	$\mathcal{E}_x$(V/m)	$\mathcal{E}_y$(V/m)
0	10cos(0) = 10	10cos(0-π/2) = 0
π/4	10cos(π/4) = 7.07	10cos(π/4-π/2) = 7.07



Note: **<u>Sense CW or righthand (RH)</u>**, axial ratio **<u>AR = 1</u>**, and tilt angle τ wrt x-direction is **<u>N/A</u>**.

Elliptical Polarization

Assume we have a UPW where the electric field is given by

$$\bar{\mathcal{E}}(z,t) = \hat{a}_x 10\cos(\omega t - \beta z) + \hat{a}_y 10\cos(\omega t - \beta z + \pi/6) \ \ \text{(V/m)}.$$

From the '$-\beta z$' term, we know the UPW is propagating in the +z-direction. On the z = 0 plane, the equation for the UPW simplifies to

$$\bar{\mathcal{E}}(0,t) = \hat{a}_x 10\cos(\omega t) + \hat{a}_y 10\cos(\omega t + \pi/6) \ \ \text{(V/m)}.$$

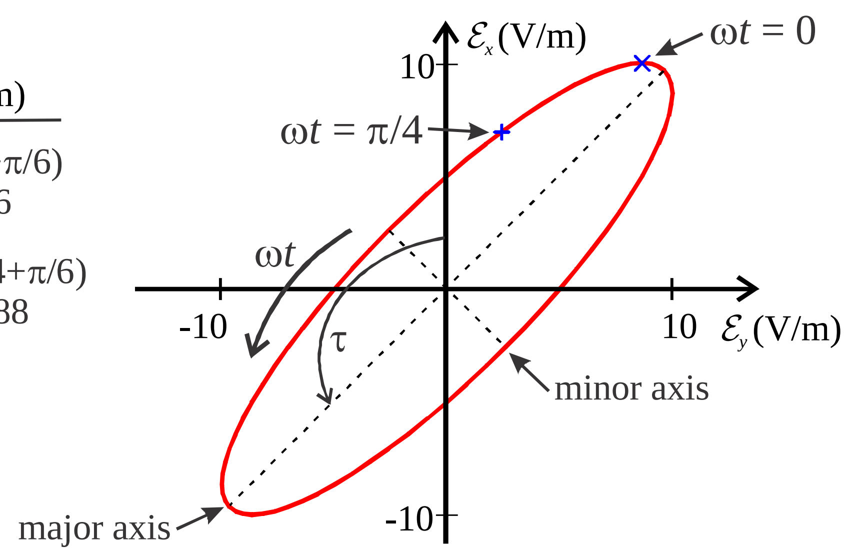
So, the electric field has two orthogonal components that are equal in magnitude **but** π/6 (30°) out of phase.

Use MathCad & CorelDraw to plot the polarization ellipse with $\mathcal{E}_x$ as the vertical axis and $\mathcal{E}_y$ as the horizontal given so that the +z-direction $(\hat{a}_x \times \hat{a}_y = \hat{a}_z)$ is into the page.

$$n := 0..72 \qquad \omega t_n := \frac{2\cdot\pi\cdot n}{72}$$

$$Ex\_ell_n := 10\cdot\cos(\omega t_n) \qquad Ey\_ell_n := 10\cdot\cos\left(\omega t_n + \frac{\pi}{6}\right)$$

ωt (rad)	$\mathcal{E}_x$ (V/m)	$\mathcal{E}_y$ (V/m)
0	10cos(0) = 10	10cos(0+π/6) = 8.66
π/4	10cos(π/4) = 7.07	10cos(π/4+π/6) = 2.588



Note: **<u>Sense CCW or lefthand (LH)</u>**, axial ratio **<u>AR ≈ 3.7</u>**, and tilt angle wrt x-direction is **<u>τ ≈ 135°</u>**.