ex. In a conductive material ($\sigma = 10^{-4}$ S/m, $\epsilon_r = 4$, & $\mu_r = 1$), the current density is $\bar{J} = \hat{a}_z \, 10 \, e^{-\alpha x} \cos(10^7 t - \beta x)$ A/m$^2$

Find $\bar{\mathcal{E}}$, $\bar{\mathcal{H}}$, $\alpha$, & $\beta$ consistant w/ Maxwell's Eqns in a region w/out free charges or convection currents.

From Ohm's Law $\quad \bar{J} = \sigma \bar{\mathcal{E}}$

$$\bar{\mathcal{E}} = \bar{J}/\sigma = \hat{a}_z \, 0.1 e^{-\alpha x} \cos(10^7 t - \beta x) \ \text{MV/m}$$

$$\hat{\bar{E}} = \hat{a}_z \, 0.1 e^{-\alpha x} e^{-j\beta x} \ \text{mV/m}$$

Use Faraday's Law (phasor form) to get $\hat{H}$ (& $\mathcal{H}$)

$$\bar{\nabla} \times \hat{\bar{E}} = -j\omega\mu \hat{\bar{H}}$$

$$\hat{H} = \frac{\left| \begin{array}{ccc|cc} \hat{a}_x & \hat{a}_y & \hat{a}_z & \hat{a}_x & \hat{a}_y \\ \partial/\partial x & \partial/\partial y & \partial/\partial z & \partial/\partial x & \partial/\partial y \\ 0 & 0 & 10^5 e^{-\alpha x} e^{-j\beta x} & 0 & 0 \end{array} \right|}{-j \, 10^7 (4\pi \times 10^{-7})}$$

## ex. cont.

$$\hat{H} = \frac{(0+0+0) - \left(\hat{a}_y 10^5(-\alpha - j\beta)\, e^{-(\alpha+j\beta)x} + 0 + 0\right)}{-j4\pi}$$

$$= \frac{\hat{a}_y\, 10^5(\alpha + j\beta)\, e^{-(\alpha+j\beta)x}}{-j4\pi}$$

$$\underline{\hat{H} = \hat{a}_y\, j\, \frac{10^5}{4\pi}(\alpha + j\beta)\, e^{-(\alpha+j\beta)x}}$$

No convection currents

Use Ampere's Law $\bar{\nabla} \times \hat{H} = (\sigma + j\omega\epsilon)\hat{E} + \cancel{\hat{J}}^{\,0}$

$$\bar{\nabla} \times \hat{H} = \left|\begin{array}{ccc|cc} \hat{a}_x & \hat{a}_y & \hat{a}_z & \hat{a}_x & \hat{a}_y \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 0 & j\frac{10^5}{4\pi}(\alpha+j\beta)e^{-(\alpha+j\beta)x} & 0 & 0 & j\frac{10^5}{4\pi}(\alpha+j\beta)e^{-(\alpha+j\beta)x} \end{array}\right.$$

$$= \left[0 + 0 + \hat{a}_z\, j\, \frac{10^5}{4\pi}(\alpha+j\beta)(-\alpha-j\beta)e^{-(\alpha+j\beta)x}\right] - \left[0+0+0\right]$$

$$\bar{\nabla} \times \hat{H} = -\hat{a}_z\, j\, \frac{10^5}{4\pi}(\alpha+j\beta)^2 e^{-(\alpha+j\beta)x} = (\sigma + j\omega\epsilon)\hat{E}$$

$$(\sigma + j\omega\epsilon)\bar{E} = \left(10^{-4} + j\,10^7(4)\epsilon_0\right]\hat{a}_z\, 10^5\, e^{-(\alpha+j\beta)x}$$

↓ divide out common terms

$$\frac{-j}{4\pi}(\alpha+j\beta)^2 = \left(10^{-4} + j\,10^7(4)\epsilon_0\right)$$

ex. cont.

$$\alpha + j\beta = \sqrt{\frac{4\pi}{-j}\left(10^{-4} + j\,10^{7}(4)\,8.854\times 10^{-12}\right)}$$

$$\underline{\alpha + j\beta = 0.009328 + j\,0.06736\ \mathrm{m}^{-1}}$$

$$\underline{\underline{\alpha = 0.009328\ \mathrm{np/m}}} \quad \text{atten. constant}$$

$$\underline{\underline{\beta = 0.06736\ \mathrm{rad/m}}} \quad \text{phase constant}$$

$$\underline{\underline{\bar{J} = \hat{a}_z\, 10\, e^{-0.00933x} \cos(10^{7}t - 0.0674x)\ \mathrm{A/m^2}}}$$

$$\underline{\bar{\mathcal{E}} = \hat{a}_z\, 0.1\, e^{-0.00933x} \cos(10^{7}t - 0.0674x)\ \mathrm{mV/m}}$$

$$\hat{\bar{H}} = \hat{a}_y\, \frac{j\,10^{5}}{4\pi}(0.00933 + j0.0674)\, e^{-0.00933x}\, e^{-j0.0674x}$$

$$= \hat{a}_y\, 541.149 \angle 172.12^\circ\, e^{-0.00933x}\, e^{-j0.0674x}$$

$$\underline{\bar{\mathcal{H}} = \hat{a}_y\, 541.15\, e^{-0.00933x} \cos(10^{7}t - 0.0674x + 172.12^\circ)\ \mathrm{A/m}}$$