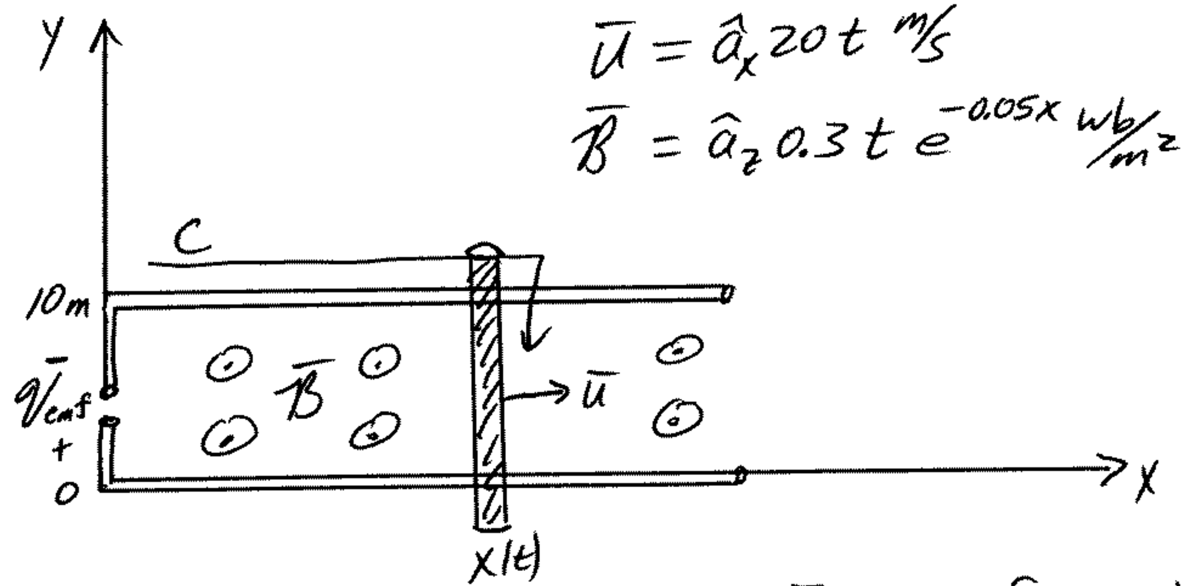
Moving loop in time-varying magnetic field example



Method 1 $\quad V_{emf} = emf = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s} + \oint_c (\bar{u} \times \bar{B}) \cdot d\bar{\ell}$

First

$$\frac{\partial \bar{B}}{\partial t} = \hat{a}_z 0.3(1) e^{-0.05x'}$$

← Note that we're ignoring the time dependence of x(t) and using "x'"

$$d\bar{s} = -\hat{a}_z dx' dy$$

$$-\int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s} = -\int_S \left(\hat{a}_z 0.3 e^{-0.05x'}\right) \cdot \left(-\hat{a}_z dx' dy\right) = \int_S 0.3 e^{-0.05x'} dx' dy$$

$$= \int_{y=0}^{10} dy \int_{x'=0}^{x(t)} 0.3 e^{-0.05x'} dx' = (y)\Big|_0^{10} \frac{0.3 e^{-0.05x'}}{-0.05}\Bigg|_{x'=0}^{x(t)}$$

$$= (10-0)(-6)\left(e^{-0.05x(t)} - e^0\right)$$

$$-\int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s} = 60\left(1 - e^{-0.05x(t)}\right)$$

ex. cont.

Method I cont.

$$d\bar{\ell} = \hat{a}_x\,dx + \hat{a}_y\,dy + \hat{a}_z\,dz$$

$$\oint_c (\bar{u} \times \bar{B}) \cdot d\bar{\ell} = \int_{y=10}^{0} \left(\hat{a}_x 20t \times \hat{a}_z 0.3t\, e^{-0.05x(t)}\right) \cdot d\bar{\ell}$$

$$+ \int 0 \cdot d\bar{\ell} \quad \leftarrow \text{rest of loop}$$

$$= \int_{y=10}^{0} -\hat{a}_y 6t^2 e^{-0.05x(t)} \cdot (\hat{a}_x\,dx + \hat{a}_y\,dy + \hat{a}_z\,dz)$$

$$= -6t^2 e^{-0.05x(t)} \int_{y=10}^{0} dy \quad \rightarrow (0-10) = -10$$

$$\oint_c (\bar{u} \times \bar{B}) \cdot d\bar{\ell} = 60\,t^2 e^{-0.05x(t)}$$

$$V_{emf} = 60\left(1 - e^{-0.05x(t)}\right) + 60t^2 e^{-0.05x(t)}$$

$$V_{emf} = 60\left[1 + (t^2 - 1)\, e^{-0.05x(t)}\right] \text{ V}$$

$$\frac{du}{dt} = 20 = a$$

what is $x(t)$? $x(t) = \cancel{x_0}^{\,0} + \cancel{u_0}^{\,0} t + \frac{1}{2} a t^2 = 10t^2$

initial ($t=0$) values

$$V_{emf} = 60\left[1 + (t^2 - 1)\, e^{-0.5t^2}\right] \text{ V} \quad t \geq 0$$

ex. cont.

Method 2 $\quad \mathcal{V}_{emf} = emf = -\dfrac{d\Psi_m}{dt}$

$$\Psi_m = \int_S \bar{B} \cdot d\bar{s} = \int_S \hat{a}_z\, 0.3t\, e^{-0.05x'} \cdot -\hat{a}_z\, dx'dy$$

$$= -0.3t \int_{y=0}^{10} dy \int_{x'=0}^{x(t)} e^{-0.05x'} dx'$$

$$= -0.3t\, (y)\Big|_0^{10} \left(\frac{e^{-0.05x'}}{-0.05}\right)\Bigg|_0^{x(t)}$$

$$= 60t\left[e^{-0.05x(t)} - e^{0}\right]$$

substitute in $x(t) = 10t^2$

$$\underline{\Psi_m = 60t\, e^{-0.5t^2} - 60t \text{ Wb}}$$

$$\mathcal{V}_{emf} = -\frac{d\Psi_m}{dt} = -\left[60\, e^{-0.5t^2} + 60t\, e^{-0.5t^2}(2)(-0.5t) - 60\right]$$

$$= -60\, e^{-0.5t^2} + 60t^2 e^{-0.5t^2} + 60$$

$$\underline{\mathcal{V}_{emf} = 60\left[1 + (t^2 - 1)\, e^{-0.5t^2}\right] \text{ V} \quad t \geq 0}$$

SAME!

Chapter 9_Faraday's Law_example (Sadiku text)

$n := 0 .. 100 \qquad t_n := \frac{n}{10} \quad s$

$$Vemf_n := 60 \cdot \left[1 + \left[(t_n)^2 - 1 \right] \cdot e^{-0.5 \cdot (t_n)^2} \right] \quad V$$