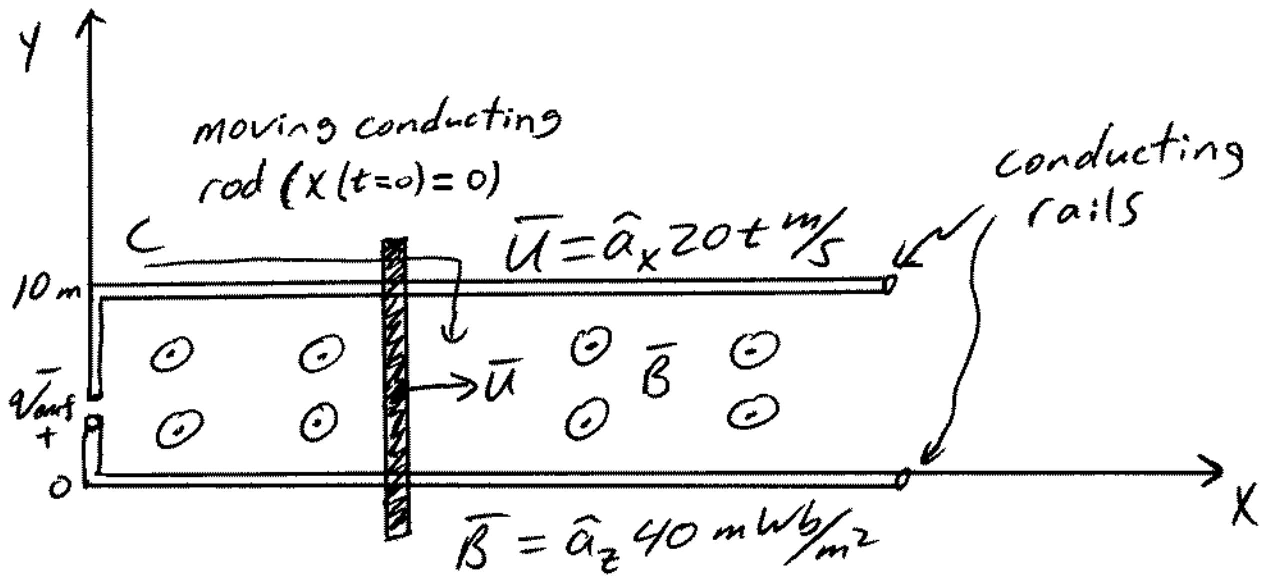
# Motional or flux-cutting emf example

## Find $\mathcal{V}_{emf}$




→ From Lenz's Law, we expect the emf to work CW around the contour to oppose the increase in $\Psi_m$ as the rod moves to the right.

→ Therefore, choose C in the CW direction and the gap voltage w/ the polarity shown (should be positive)

## Method 1

$$\mathcal{V}_{emf} = emf = \oint_C \bar{\mathcal{E}}_m \cdot d\bar{\ell} = \oint_C (\bar{u} \times \bar{B}) \cdot d\bar{\ell}$$

moving rod $\bar{u} \times \bar{B} = \hat{a}_x 20t \times \hat{a}_z 40 \times 10^{-3} = -\hat{a}_y 0.8t$

elsewhere $\bar{u} \times \bar{B} = 0$

## Method 1 cont.

(moving CW around loop; 0 · dl elsewhere)

$$\mathcal{V}_{emf} = \int_{y=10m}^{0} (-\hat{a}_y 0.8t) \cdot (dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z) + \int 0 \cdot d\bar{l}$$

$$= -0.8t \int_{y=10}^{0} dy = -0.8t\,(y)\Big|_{10}^{0} = -0.8t\,(0-10)$$

$$\mathcal{V}_{emf} = +8t \text{ V}$$

## Method 2

$$\mathcal{V}_{emf} = emf = -\frac{d\Psi_m}{dt}$$

(use RHR w/ C)

$$\Psi_m = \int_S \bar{B} \cdot d\bar{s} = \int_S \bar{B} \cdot -d\bar{s}_z$$

$$= \int_S \hat{a}_z 40\times10^{-3} \cdot -\hat{a}_z\,dx\,dy = -0.04 \int_{y=0}^{10m} dy \int_{x=0}^{x(t)} dx$$

$$= -0.04\,(10-0)(x(t)-0)$$

From physics, $x(t) = x_o + u_o t + \frac{1}{2} a t^2 = \frac{1}{2}(20)t^2$ (initial velocity; acceleration $= \frac{d\bar{u}}{dt}$)

$$\Psi_m = -0.04(10)(10t^2) = -4t^2 \text{ (wb)}$$

$$\mathcal{V}_{emf} = -\frac{d\Psi_m}{dt} = -(-4t)(2) = +8t \text{ V}$$

← Same!