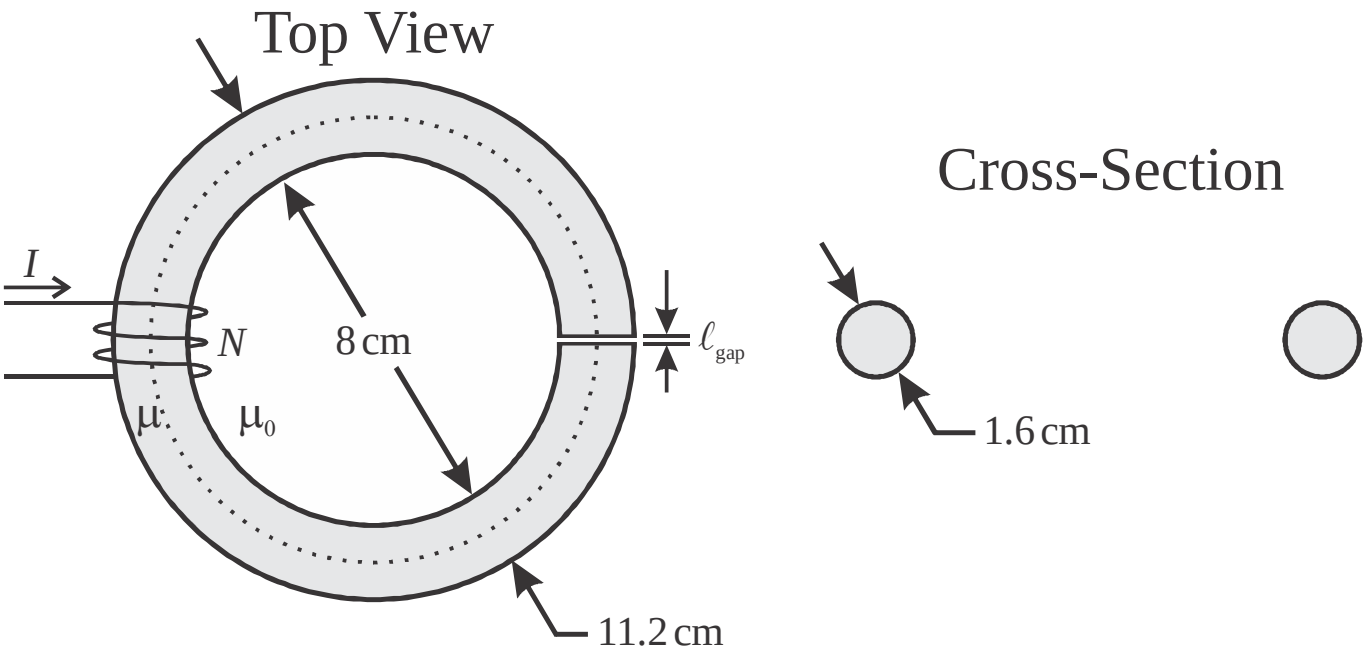
Magnetic Circuit Example

One hundred turns of wire carrying 0.5 A of current is wrapped around a toroid with an inner diameter of 8 cm and outer diameter of 11.2 cm and having circular cross-section. The toroid was made by bending a cobalt ($\mu = 250\mu_0$) rod around a circular form, leaving a 1.5 mm gap. Use magnetic circuits to determine the various magnetic quantities related to this problem first neglecting fringing and then estimating the effects of fringing at the gap.



No fringing (i.e., $S_{core} = S_{gap}$)

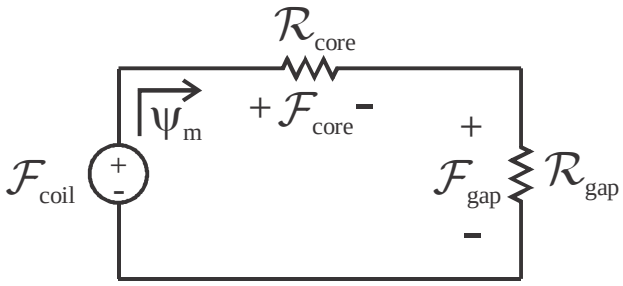
$$\mathcal{R}_{core} = \frac{\ell_{core}}{\mu_{core} S_{core}} = \frac{\pi d_{ave} - \ell_{gap}}{\mu_{core}(\pi r_{core}^2)} = \frac{\pi(9.6\bullet10^{-2}) - 1.5\bullet10^{-3}}{250(4\pi\bullet10^{-7})[\pi(0.8\bullet10^{-2})^2]}$$

$$= \frac{0.300092895}{250(4\pi\bullet10^{-7})[0.000201062]} = \underline{4,750,899.5 \ \mathrm{A\cdot t/Wb}}$$

$$\mathcal{R}_{gap} = \frac{\ell_{gap}}{\mu_0 S_{gap}} = \frac{\ell_{gap}}{\mu_0 S_{core}} = \frac{1.5\bullet10^{-3}}{4\pi\bullet10^{-7}[0.000201062]}$$

$$= \underline{5,936,786 \ \mathrm{A\cdot t/Wb}}$$

$$\mathcal{F}_{coil} = NI = 100(0.5) = \underline{50 \ \mathrm{A\cdot t}}$$



$$\psi_m = \frac{\mathcal{F}_{\text{coil}}}{\mathcal{R}_{\text{core}} + \mathcal{R}_{\text{gap}}} = \frac{50}{4750899.5 + 5936786} = 4.67828 \cdot 10^{-6}\ \text{Wb} = \underline{4.678\ \mu\text{Wb}}$$

$$B_{\text{gap}} = B_{\text{core}} = \frac{\psi_m}{S_{\text{core}}} = \frac{4.67828 \cdot 10^{-6}}{0.000201062} = 0.023268\ \text{Wb}/\text{m}^2 = \underline{23.268\ \text{mT}}$$

$$\mathcal{F}_{\text{core}} = \psi_m \mathcal{R}_{\text{core}} = 4.67828 \cdot 10^{-6}(4750899.5) = \underline{22.226\ \text{A} \cdot \text{t}} \qquad \text{OR}$$

$$= \mathcal{F}_{\text{coil}} \frac{\mathcal{R}_{\text{core}}}{\mathcal{R}_{\text{core}} + \mathcal{R}_{\text{gap}}} = 50 \frac{4750899.5}{4750899.5 + 5936786} = \underline{22.226\ \text{A} \cdot \text{t}}$$

$$\mathcal{F}_{\text{gap}} = \psi_m \mathcal{R}_{\text{gap}} = 4.67828 \cdot 10^{-6}(5936786) = \underline{27.774\ \text{A} \cdot \text{t}} \qquad \text{OR}$$

$$= \mathcal{F}_{\text{coil}} \frac{\mathcal{R}_{\text{gap}}}{\mathcal{R}_{\text{core}} + \mathcal{R}_{\text{gap}}} = 50 \frac{5936786}{4750899.5 + 5936786} = \underline{27.774\ \text{A} \cdot \text{t}}$$

$$H_{\text{core}} = \frac{B_{\text{core}}}{\mu_{\text{core}}} = \frac{0.023268}{250(4\pi \bullet 10^{-7})} = \underline{74.064\ \text{A}/\text{m}} \qquad \text{OR}$$

$$= \frac{\mathcal{F}_{\text{core}}}{\ell_{\text{core}}} = \frac{22.226}{0.300092895} = \underline{74.064\ \text{A}/\text{m}}$$

$$H_{\text{gap}} = \frac{B_{\text{gap}}}{\mu_0} = \frac{0.023268}{4\pi \bullet 10^{-7}} = \underline{18{,}516\ \text{A}/\text{m}} \qquad \text{OR}$$

$$= \frac{\mathcal{F}_{\text{gap}}}{\ell_{\text{gap}}} = \frac{27.774}{1.5 \bullet 10^{-3}} = \underline{18{,}516\ \text{A}/\text{m}}$$

$$L_{\text{coil}} = \frac{N\psi_m}{I} = \frac{100(4.67828 \cdot 10^{-6})}{0.5} = 0.000935656\ \text{H} = \underline{0.93566\ \text{mH}} \quad \text{OR}$$

$$= \frac{N^2}{\mathcal{R}_{\text{eq}}} = \frac{N^2}{\mathcal{R}_{\text{core}} + \mathcal{R}_{\text{gap}}} = \frac{100^2}{4750899.5 + 5936786} = \underline{0.93566\ \text{mH}}$$

<u>w/ fringing</u> (i.e., $S_{\text{gap}} > S_{\text{core}}$**)**

$\mathcal{R}_{\text{core}} = \underline{4{,}750{,}899.5 \ \text{A}\cdot\text{t}/\text{Wb}}$ and $\mathcal{F}_{\text{coil}} = \underline{50 \ \text{A}\cdot\text{t}}$ (unchanged)

$$\mathcal{R}_{\text{gap}} = \frac{\ell_{\text{gap}}}{\mu_0 S_{\text{gap}}} = \frac{\ell_{\text{gap}}}{\mu_0 \pi (r_{\text{core}} + \ell_{\text{gap}})^2} = \frac{1.5\text{•}10^{-3}}{4\pi\text{•}10^{-7}[\pi(0.8\text{•}10^{-2} + 1.5\text{•}10^{-3})^2]}$$

$$= \frac{1.5\text{•}10^{-3}}{4\pi\text{•}10^{-7}(0.000283529)} = \underline{4{,}210{,}021.5 \ \text{A}\cdot\text{t}/\text{Wb}}$$

Note: $\dfrac{S_{\text{gap}}}{S_{\text{core}}} = \dfrac{0.000283529}{0.000201062} = 1.41$

$$\psi_m = \frac{\mathcal{F}_{\text{coil}}}{\mathcal{R}_{\text{core}} + \mathcal{R}_{\text{gap}}} = \frac{50}{4750899.5 + 4210021.5} = 5.57978\cdot10^{-6} \ \text{Wb} = \underline{5.5798 \ \mu\text{Wb}}$$

$$B_{\text{core}} = \frac{\psi_m}{S_{\text{core}}} = \frac{5.57978\cdot10^{-6}}{0.000201062} = 0.0277516 \ \text{Wb}/\text{m}^2 = \underline{27.7516 \ \text{mT}}$$

$$B_{\text{gap}} = \frac{\psi_m}{S_{\text{gap}}} = \frac{5.57978\cdot10^{-6}}{0.000283529} = 0.0196798 \ \text{Wb}/\text{m}^2 = \underline{19.6798 \ \text{mT}}$$

$$\mathcal{F}_{\text{core}} = \psi_m \mathcal{R}_{\text{core}} = 5.57978\cdot10^{-6}(4750899.5) = \underline{26.509 \ \text{A}\cdot\text{t}}$$

$$\mathcal{F}_{\text{gap}} = \psi_m \mathcal{R}_{\text{gap}} = 5.57978\cdot10^{-6}(4210021.5) = \underline{23.491 \ \text{A}\cdot\text{t}}$$

$$H_{\text{core}} = \frac{B_{\text{core}}}{\mu_{\text{core}}} = \frac{0.0277516}{250(4\pi\text{•}10^{-7})} = \underline{88.336 \ \text{A}/\text{m}}$$

$$H_{\text{gap}} = \frac{B_{\text{gap}}}{\mu_0} = \frac{0.0196798}{4\pi\text{•}10^{-7}} = \underline{15{,}660.7 \ \text{A}/\text{m}}$$

$$L_{\text{coil}} = \frac{N\psi_m}{I} = \frac{100(5.5798\cdot10^{-6})}{0.5} = 0.001115957 \ \text{H} = \underline{1.116 \ \text{mH}}$$