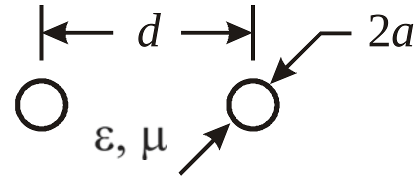
EE 382 Lossy Transmission Line Example Problem



$d := 4\cdot10^{-2}$ m $a := \frac{0.5\cdot10^{-2}}{2}$ m

$f := 500\cdot10^{6}$ Hz $\omega := 2\cdot\pi\cdot f$

$\sigma\_cu := 5.8\cdot10^{7}$ S/m $\sigma\_ins := 0.001$ S/m $\mu0 := 4\cdot\pi\cdot10^{-7}$ H/m

$\varepsilon0 := 8.8541878\cdot10^{-12}$ F/m $\varepsilon r\_ins := 1.9$

Calculate lossy transmission line parameters

$\delta := \frac{1}{\sqrt{\pi\cdot f\cdot\mu0\cdot\sigma\_cu}}$ $\boxed{\delta = 2.95543\times10^{-6}}$ m, skin depth

$R := \frac{1}{\pi\cdot a\cdot\delta\cdot\sigma\_cu}$ $\boxed{R = 0.74278}$ Ω/m, resistance-per-unit-length

$L := \frac{\mu0}{\pi}\cdot\operatorname{acosh}\left(\frac{d}{2\cdot a}\right)$ $\boxed{L = 1.10746\times10^{-6}}$ H/m, inductance-per-unit-length

$G := \frac{\pi\cdot\sigma\_ins}{\operatorname{acosh}\left(\frac{d}{2\cdot a}\right)}$ $\boxed{G = 1.1347\times10^{-3}}$ S/m, conductance-per-unit-length

$C := \frac{\pi\cdot\varepsilon r\_ins\cdot\varepsilon0}{\operatorname{acosh}\left(\frac{d}{2\cdot a}\right)}$ $\boxed{C = 1.9089\times10^{-11}}$ F/m, capacitance-per-unit-length

$\gamma := \sqrt{(R + j\cdot\omega\cdot L)\cdot(G + j\cdot\omega\cdot C)}$ $\boxed{\gamma = 0.13819 + 14.44525i}$ 1/m, propagation constant

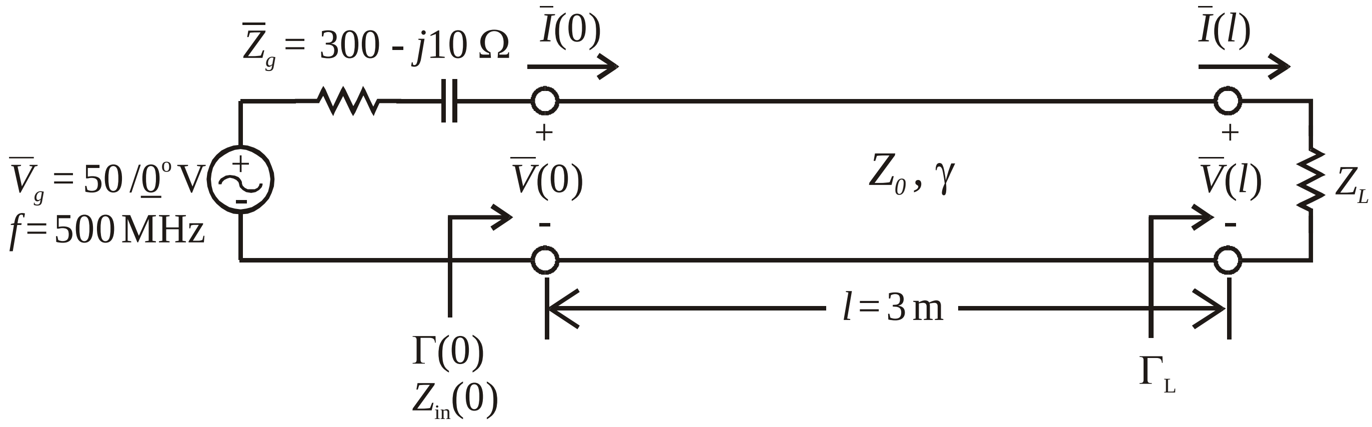
$\alpha := \operatorname{Re}(\gamma)$ $\boxed{\alpha = 0.1382}$ np/m, atten. constant or $20\log\left(e^{\alpha}\right) = 1.2$ dB/m $\beta := \operatorname{Im}(\gamma)$

$\boxed{\beta = 14.445}$ rad/m, phase constant $u := \frac{\omega}{\beta}$ $\boxed{u = 2.17483\times10^{8}}$ m/s, phase velocity

$\lambda := \frac{2\cdot\pi}{\beta}$ $\boxed{\lambda = 0.43497}$ m, wavelength

$Z0 := \sqrt{\frac{(R + j\cdot\omega\cdot L)}{(G + j\cdot\omega\cdot C)}}$ $\boxed{Z0 = 240.8328 + 2.2525i}$ Ω, characteristic impedance

Lossy transmission line circuit



$Zg := 300 - j \cdot 10 \quad \Omega \qquad ZL := 300 + j \cdot 0 \quad \Omega \qquad l := 3.0 \quad m \qquad Vg := 50 \quad V$

$$\Gamma L := \frac{ZL - Z0}{ZL + Z0} \qquad \Gamma g := \frac{Zg - Z0}{Zg + Z0}$$

$\Gamma L = 0.1094 - 4.6204i \times 10^{-3}$ $\quad |\Gamma L| = 0.1095 \quad$ $\arg(\Gamma L) \cdot \frac{180}{\pi} = -2.4188$ deg

$\Gamma g = 0.1097 - 0.0211i$ $\quad |\Gamma g| = 0.1117 \quad$ $\arg(\Gamma g) \cdot \frac{180}{\pi} = -10.8789$ deg

$$\Gamma 0 := \Gamma L \cdot e^{-2 \cdot \gamma \cdot l} \qquad Zin0 := Z0 \cdot \frac{(1 + \Gamma 0)}{(1 - \Gamma 0)}$$

$\Gamma 0 = 0.015 + 0.0454i$ $\quad |\Gamma 0| = 0.0478 \quad$ $\arg(\Gamma 0) \cdot \frac{180}{\pi} = 71.6711$ deg

$Zin0 = 246.9354 + 24.7814i \ \Omega$ $\quad |Zin0| = 248.1758 \ \Omega \quad$ $\arg(Zin0) \cdot \frac{180}{\pi} = 5.73$ deg

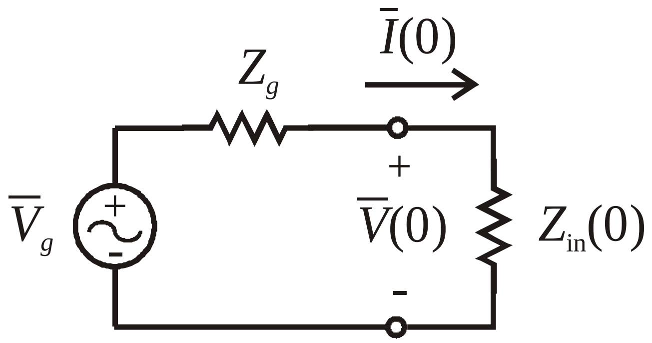
OR

$$Zin := Z0 \cdot \frac{(ZL + Z0 \cdot \tanh(\gamma \cdot l))}{Z0 + ZL \cdot \tanh(\gamma \cdot l)}$$

$Zin = 246.9354 + 24.7814i \quad \Omega$ **SAME!**

Notice how Zin is converging toward Z0 = 240.83 + j2.25 Ohms

Now, we can draw the equivalent circuit seen by the generator.



Use simple circuit theory to calculate the input voltage, current, and power.

$I0 := \dfrac{Vg}{Zg + Zin}$ $|I0| = 0.0914$ A $\arg(I0)\cdot\dfrac{180}{\pi} = -1.5481$ deg

$V0 := \dfrac{Vg\cdot Zin}{Zg + Zin}$ $|V0| = 22.6796$ V $\arg(V0)\cdot\dfrac{180}{\pi} = 4.1827$ deg

$P0 := 0.5\cdot \mathrm{Re}\left(V0\cdot\overline{I0}\right)$ $P0 = 1.03111$ W **Power <u>into</u> transmission line.**

Next, we'll find how much power makes it to the load and how much is lost.

$IL := \dfrac{Vg}{Z0 + Zg}\cdot e^{-\gamma\cdot l}\cdot\dfrac{1 - \Gamma L}{1 - \Gamma g\cdot\Gamma L\cdot e^{-2\gamma\cdot l}}$ $|IL| = 0.0545$ A $\arg(IL)\cdot\dfrac{180}{\pi} = 38.431$ deg

$VL := \dfrac{Vg\cdot Z0}{Z0 + Zg}\cdot e^{-\gamma\cdot l}\cdot\dfrac{1 + \Gamma L}{1 - \Gamma g\cdot\Gamma L\cdot e^{-2\gamma\cdot l}}$ $|VL| = 16.359$ V $\arg(VL)\cdot\dfrac{180}{\pi} = 38.431$ deg

$PL := 0.5\cdot \mathrm{Re}\left(VL\cdot\overline{IL}\right)$ $PL = 0.44604$ W **Power <u>delivered</u> to load.**

$Plost := P0 - PL$ $Plost = 0.58507$ W **Power <u>lost</u> in transmission line.**

Note that over half the power was lost in the transmission line.

<u>OR</u>, alternate methods to finding load current, voltage, & power

$V0\_fwd := \frac{V0}{1 + \Gamma 0}$ $\quad V0\_fwd = 22.31261 + 0.63268i \ \ V$

$V0\_fwd2 := 0.5\cdot(V0 + Z0\cdot I0)$ $\quad V0\_fwd2 = 22.31261 + 0.63268i \ \ V$

<u>ALL THE SAME!!</u>

$V0\_fwd3 := 0.5\cdot(VL + Z0\cdot IL)\cdot e^{\gamma\cdot l}$ $\quad V0\_fwd3 = 22.31261 + 0.63268i \ \ V$

$|V0\_fwd| = 22.3216 \ \ V$ $\quad \arg(V0\_fwd)\cdot\frac{180}{\pi} = 1.6242 \ \ deg$

Knowing the magnitude of the forward traveling wave, we can now use the phasor voltage and current equations, evaluated at *z = l*.

$IL2 := \frac{V0\_fwd}{Z0}\cdot e^{-\gamma\cdot l}\cdot(1 - \Gamma L)$ $\quad |IL2| = 0.0545 \ \ A$ $\quad \arg(IL2)\cdot\frac{180}{\pi} = 38.431 \ \ deg$

$VL2 := V0\_fwd\cdot e^{-\gamma\cdot l}\cdot(1 + \Gamma L)$ $\quad |VL2| = 16.359 \ \ V$ $\quad \arg(VL2)\cdot\frac{180}{\pi} = 38.431 \ \ deg$

$PL2 := 0.5\cdot Re\left(VL2\cdot\overline{IL2}\right)$ $\quad PL2 = 0.44604 \ \ W$ **<u>SAME AS BEFORE!!</u>**