13.1 Introduction

- I'll be drawing heavily on outside resources, e.g., my own notes, *Antenna Theory, Analysis and Design (Fourth Edition)* by C. Balanis, etc.

Definition - That part of a transmitting or receiving system that is designed to radiate or to receive electromagnetic waves. (IEEE Std. 145-1993).

Types of Antennas

Wire Antennas:

- Cheap, Reliable


Car (whip/monopole)


TV [Loop (UHF) + "bunny ears"/dipole (VHF))


Helix (Space comm.)

Aperture Antennas:

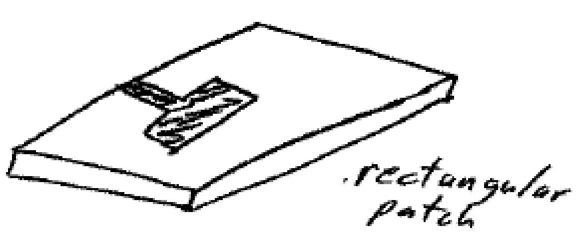
- Rugged, High Gains

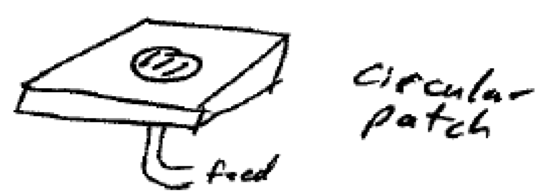
Horns (Dish Feeds)


Slotted Waveguides (Flush Mounted - military)

Microstrip Antennas:

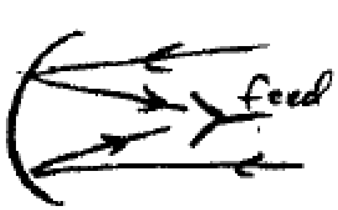
- Cheap + easy to manufacture



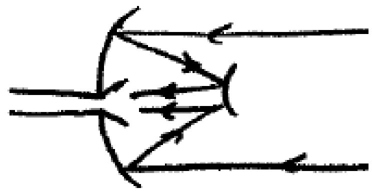


Reflector Antennas

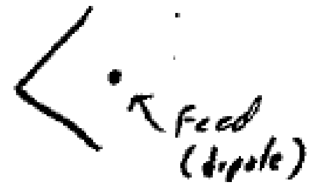
- Very common for space applications
- Fed by other antenna
- Can achieve very large gains



Parabolic Dish



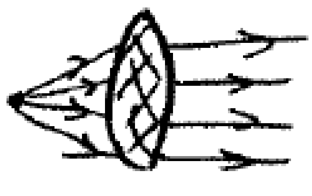
w/ Cassegrain feed



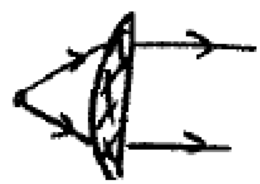
Corner reflector
(side view)

Lens Antennas

- Not very common



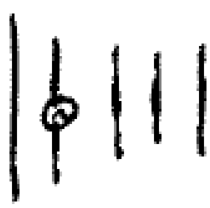
Convex-convex



Convex-plane

Arrays

- Use more than one antenna to achieve design goal
- More flexibility to get desired radiation pattern, beam steering…



Yagi-Uda Array

Slotted Waveguide

<u>Radiation Mechanism</u>

How is radiation accomplished? I.e., How do we take a confined wave/field in a transmission line or waveguide and "detach" it to form a wave propagating in free space?

For radiation to occur, we must have a time-varying current or an acceleration (deceleration) of charge.

Examples-

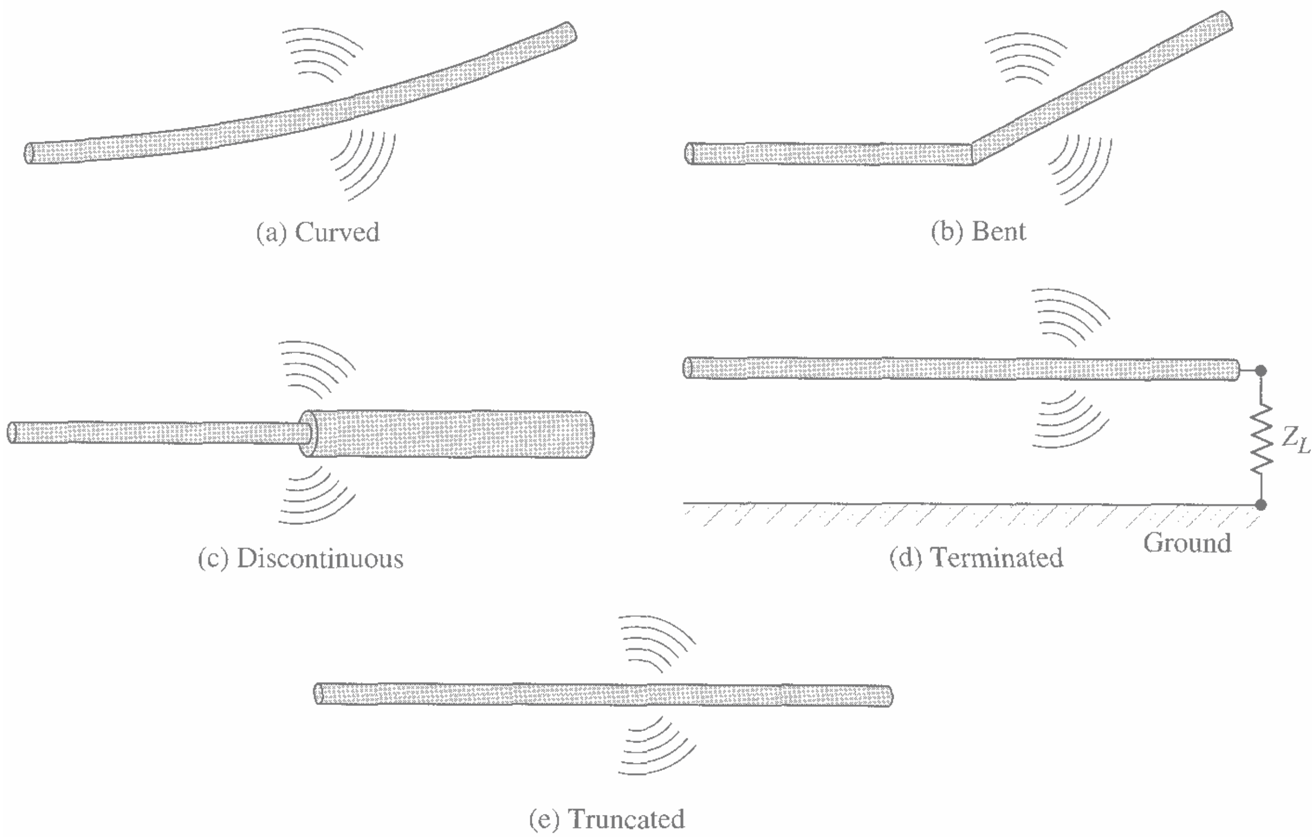
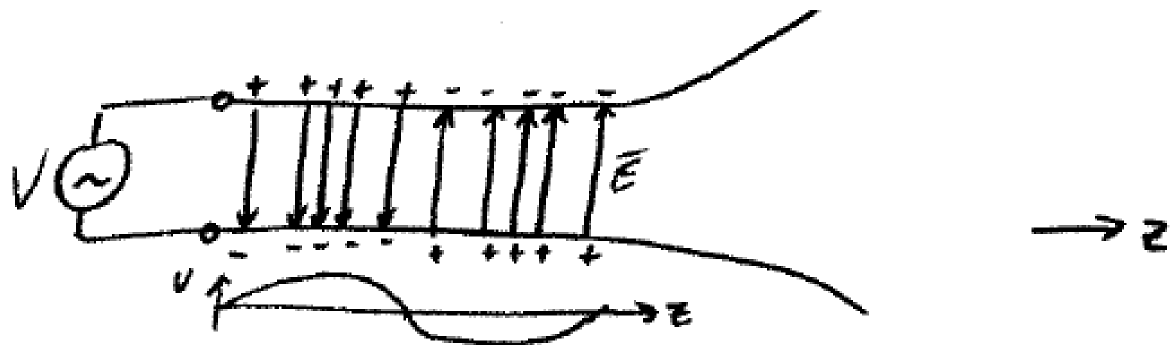


Figure 1.10 Wire configurations for radiation [Balanis, 4th edn]

Consequences

1. No charge movement → no current → no radiation
2. Uniform charge velocity (speed + direction)
   a) No radiation if wire is straight + infinitely long
   b) Radiation only if Figure 1.10 above conditions met
3. If charge is oscillating (e.g. sinusoidal excitement), it radiates even if wire is straight.

Now let's consider how waves are radiated, using a two-wire example.



1) A voltage source creates an electric field between the conductors that propagates down the transmission line.

   → Electric field lines act on free electrons so that they start on + charges and end on - charges.

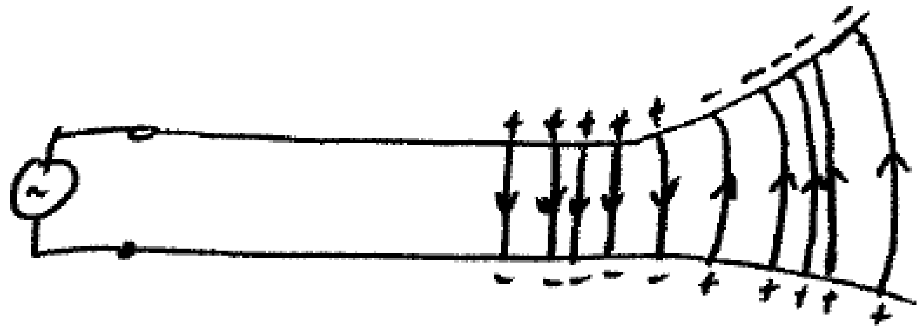
   → Remember electric field lines can:
      1) Start on + charges and end on - charges.
      2) Start on + charges and end at infinity.
      3) Start at infinity and end on - charges.
      4) Form closed loops (no charges involved).

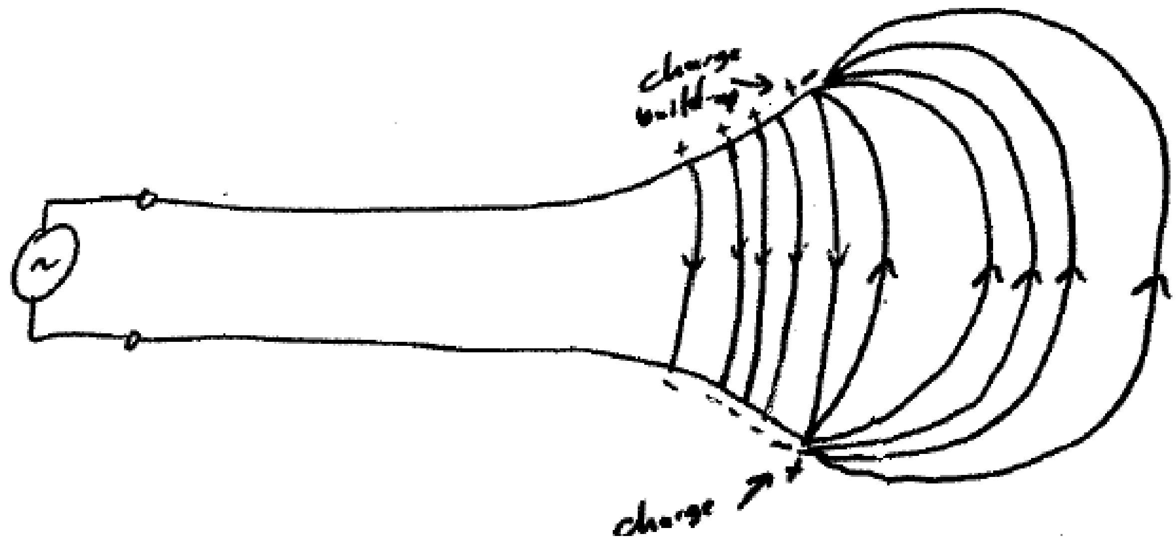
   → The movement of charges induces a magnetic field.

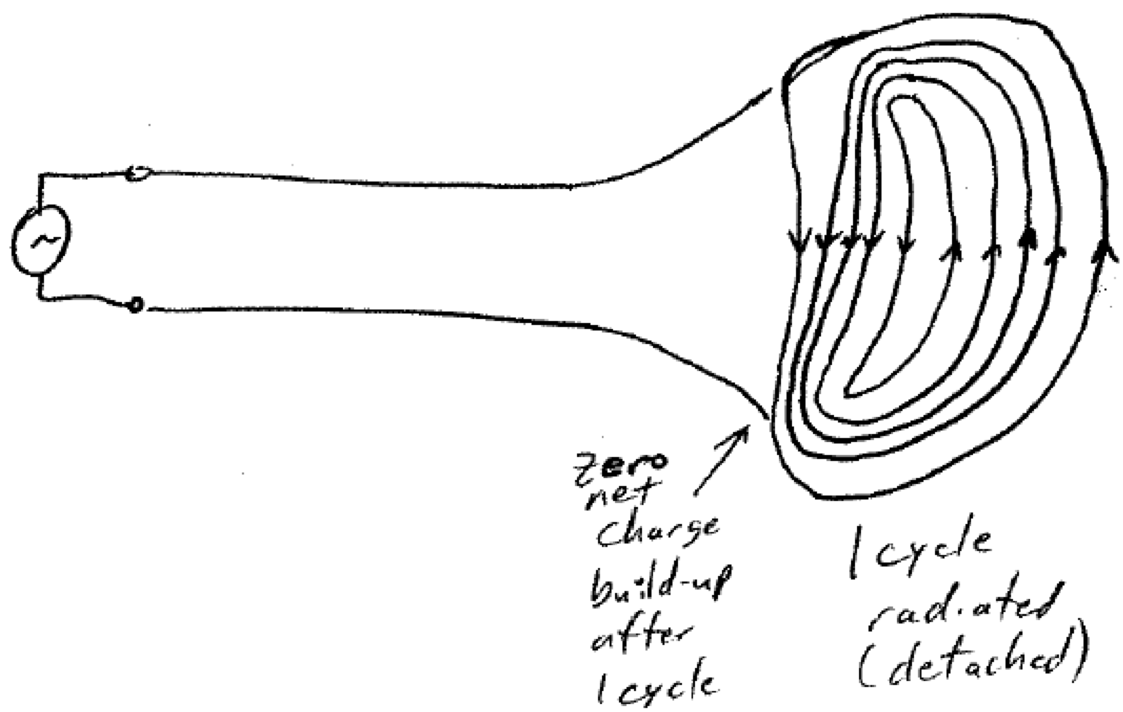
   → Magnetic field lines are always closed loops, no known physical magnetic charges. [Note: Non-physical magnetic charges and current are sometimes used for mathematical convenience.]

2) Note that if the voltage source were to turn off, the electric/E and magnetic/H fields already created would continue to exist and be radiated. (Stone in pond analogy)

3) Let the electric field continue to progress down the transmission line and antenna. For clarity, only a single cycle is shown.







Abbreviated History

Maxwell → Maxwell's Equations - 1873.
→ Radiated waves are electromagnetic.

Hertz → 1886 demonstrated first wireless electromagnetic radiation (used spark gap generator, dipole and loop antennas).

Marconi → 1901 achieved transatlantic wireless transmission.

1900-1940's → Most antenna work focused on wire antennas up to UHF (470- 890 MHz) and related electronics.

WWII years → MIT Radiation Lab. (huge burst of theoretical as well as practical research)
→ Aperture antennas. (horns, waveguide slots, reflectors…)
→ High power RF/microwave sources such as klystron and magnetron developed.

Late 1940's-50's → Frequency independent antennas. E.g., LPDA, …)
→ Helical antennas.

1960's -present → huge impact of computers making numerical methods practical (e.g. MoM, FOTO…)

13.6 Antenna Characteristics

- Definitions of various parameters are needed to describe performance of antennas.

IEEE Defn – The spatial distribution of a quantity that characterizes the electromagnetic field generated by an antenna.

→ Patterns are typically for the far-field region (far enough away from antenna that reactive fields are negligible) and are normalized by the maximum value of the quantity in question

→ Patterns can either be 3D or 2D

→ 3D plots are nice for visualizing the patterns, but not as useful for reading off actual values

→ Some popular quantities for plotting are

$|E|$, $\frac{|E|}{|E|_{max}}$, U ~ radiation intensity ($\propto$ Power), D ~ directivity

G ~ Gain, ...

→ 2D plots can be in cartesian/rectangular or polar formats

Often radiation patterns are plotted with quantities expressed in decibels (dB).
For this to be possible, the argument of the $\log_{10}()$ function must be <u>unitless</u> + in terms of power.

ex. $P \propto |E|^2$ $10\log_{10}\left(\frac{|E|^2}{|E|_{max}^2}\right) = 20\log_{10}\left(\frac{|E|}{|E|_{max}}\right)$

$P \propto |H|^2$ $10\log_{10}\left(\frac{|H|^2}{|H|_{max}^2}\right) = 20\log_{10}\left(\frac{|H|}{|H|_{max}}\right)$

normalized

<u>or</u> quantities can be referenced to a fixed value

ex. $20\log_{10}\left(\frac{|E|}{1\,\mu V/m}\right) \Rightarrow dB\mu V$ or dB wrt $1\mu V$

$10\log_{10}\left(\frac{P}{1mW}\right) \Rightarrow dBm$ or dB wrt 1mW

$10\log_{10}\left(\frac{P}{1W}\right) \Rightarrow dBW$ or dB wrt 1W

How do we generate these plots?
→ MatLab, Math Cad, Excel, ... (see examples)

Plotting antenna radiation patterns:

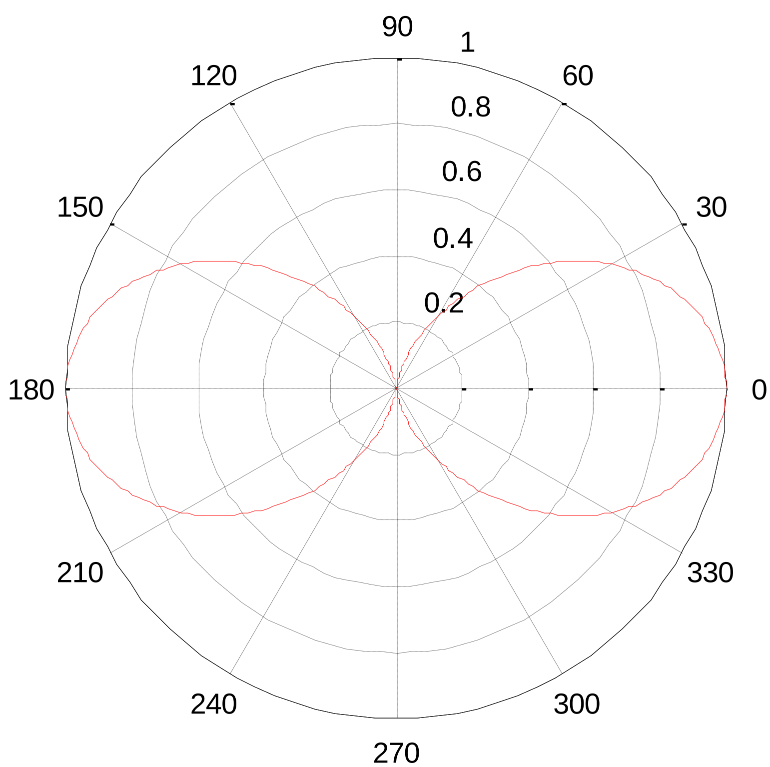
polar.m from MATLAB:

```
>> help polar

 POLAR  Polar coordinate plot.
    POLAR(THETA, RHO) makes a plot using polar coordinates of
    the angle THETA, in radians, versus the radius RHO.
    POLAR(THETA,RHO,S) uses the linestyle specified in string S.
    See PLOT for a description of legal linestyles.

    See also PLOT, LOGLOG, SEMILOGX, SEMILOGY.
```

Example:

(From MATLAB Command Window)

```
>> ang1 = 0:1:359;                              % angles in degrees
>> rho1 = cos(ang1*pi/180).*cos(ang1*pi/180);  % radial values
>> polar(ang1*pi/180,rho1,'r-')                 % plot (converted angles to radians)
```



Notes: These plots are strictly linear and radial values **must** be positive.

radpat.m found on course webpage:

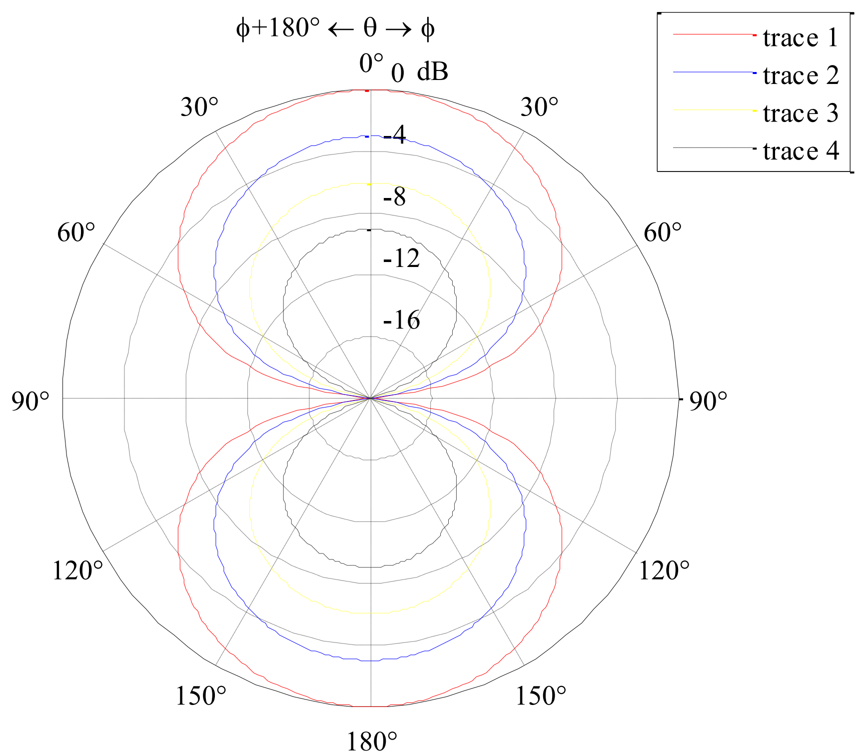
```
function radpat(ang1,R1,st1,ang2,R2,st2,ang3,R3,st3,ang4,R4,st4)
%RADPAT  Polar coordinate plot used for antenna radiation patterns.
%       RADPAT(ANG1,R1,ST1,ANG2,R2,ST2,ANG3,R3,ST3,ANG4,R4,ST4)
%         plots up to four curves in dB format.
%
%       ANGi are angles in degrees,
%       Ri are radiation pattern values (radii for plot traces), &
%       STi are the linestyles.
%       See PLOT for a description of legal linestyles.
%
%       Ri can be in dB or not in dB (resulting plot is in dB).
%       Axis labels can be placed on horizontal or vertical axis.
%       Choice of normalized or unnormalized (show gains) patterns.
%       Minimum dB level at plot center can be specified.
%       Maximum dB level at outermost plot circle can be specified for unnormalized patterns.
%       Line width of radiation patterns can be specified.
%       Legend can be placed. To move the legend, press left mouse button on the legend and
%         drag to the desired location.
%       Grid linetype can be specified.
%       Default values are inside [], press Enter to chose default.
%       0 degrees can be at North/Top or East/Right side of plot.
%
%       Example:  radpat(a1,r1,'r-',a2,r2,'y--')
%
%       Based on polarpat.m by D. Liu, 9/13/1996.
%       T.J. Watson Research center, IBM
%       P.O.Box 218
%       Yorktown Heights, NY 10598
%       Email: dliu@watson.ibm.com
%
%       Updated by Thomas P. Montoya, SDSM&T, 1/23/2006
%       * allow up to four traces
%       * added degree symbols to plot spoke labels
%       * for plots vs. theta keep spoke labels in 0 to +180 deg
%         range and indicate that negative theta angles are for
%         phi+180 deg and
%       * orient plot so that 0 degrees at the top (North)
```

Note: The resulting radiation pattern plot is in dB regardless of whether the input variable(s) (e.g., rho1) is originally in dB or not.

Example:

(From MATLAB Command Window)

```
>> ang = 0:1:359;                                  % Define angles in degrees
>> rho1 = cos(ang1*pi/180).*cos(ang1*pi/180);     % Define radiation patterns
>> rho2 = 0.5*rho1;
>> rho3 = 0.5*rho2;
>> rho4 = 0.5*rho3;
>> radpat(ang,rho1,'r-',ang,rho2,'b-',ang,rho3,'y-.',ang,rho4,'k--')
Are input values in dB (Y/N)[Y]? n
Normalize to the Maximum Gain Value (Y/N)[Y]? y
Minimum dB value at plot center [-40]? -20
Are the angles theta values? (Y/N)[Y]? y
Labels on Vertical or Horizontal axis (V/H)[V]? v
Pattern line width [1.25]:
Legend for traces on graph (Y/N)[N]? y
Enter label for trace 1: trace 1
Enter label for trace 2: trace 2
Enter label for trace 3: trace 3
Enter label for trace 4: trace 4
Put a box around the legend (Y/N)[Y]?
Line type of grid(-, --, -., :)[:]?
```



Notes: You may need to move labels around on the MatLab figure window using the mouse (click arrow icon, then left click and drag with mouse).

polarpat.m found on internet & course webpage:

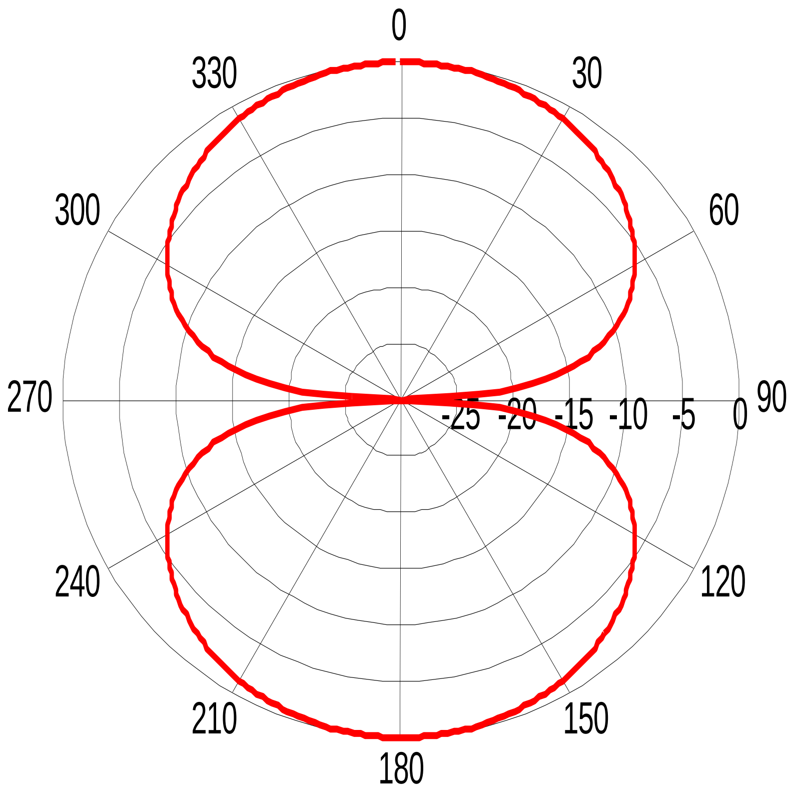
```
function polarpat(ang1,rho1,st1,ang2,rho2,st2,ang3,rho3,st3)
% POLARPAT  Polar coordinate plot used for antenna radiation patterns.
%       POLARPAT(ANG1,RHO1,ST1,ANG2,RHO2,ST2,ANG3,RHO3,ST3) plots up to
%       three curves. ANGi is angles in degress, RHOi is radius, and
%       STi is linestyle.
%       RHOi can be in dB or not in dB.
%       Axis labels can be placed horzontally or vertically.
%       Choice of normalized or unnormalized (showing gains) patterns.
%       Minimum level at the polar center can be specified.
%       Maximum level at the polar outmost circle can be specified for
%         unnormalized patterns.
%       Line width of radiation patternns can be specified.
%       Legend can be placed. To move the legend, press the left mouse
%         button on the legend and drag to the desired location.
%       Grid linetypes can be specified.
%       Default value is inside [], press Enter to chose default.
%       See PLOT for a description of legal linestyles.
%       0 degree can be in the East or North direction.
%       Example:  polarpat(a1,r1,'r-',a2,r2,'y--')
%       Written by Duixian Liu, on September 13, 1996.
%       T.J. Watson Research center
%       IBM
%       P.O.Box 218
%       Yorktown Heights, NY 10598
%       Email: dliu@watson.ibm.com
…
```

Note: The resulting radiation pattern plot is in dB regardless of whether the input variable(s) (e.g., rho1) is originally in dB or not.

Example:

(From MATLAB Command Window)

```
>> ang1 = 0:1:359;          % define angles in degrees
>> rho1 = cos(ang1*pi/180).*cos(ang1*pi/180);
>> polarpat(ang1,rho1,'r-')
Are input values in dB (Y/N)[Y]? N
Normalize to the Maximum Gain Value (Y/N)[Y]? Y
The minimum dB value at polar center [-50]? -30
Put axis label Vertically or Horizontally (V/H)[H]?
Pattern line width [1.0]: 1
Is 0 degree in the North or East (N/E)[E]? N
Line type of grid(-, --, -., :)[-]? -
>>
```



Notes: You may need to move labels around on the MatLab figure window using the mouse (click arrow icon, then left click and drag with mouse).

λ/2 Dipole directivity radiation patterns- MathCad

Spherical coordinates

$N := 50 \qquad m := 0..N \qquad n := 0..N$

$$\theta_m := \frac{\pi \cdot m}{N} \qquad \phi_n := \frac{2 \cdot \pi \cdot n}{N}$$

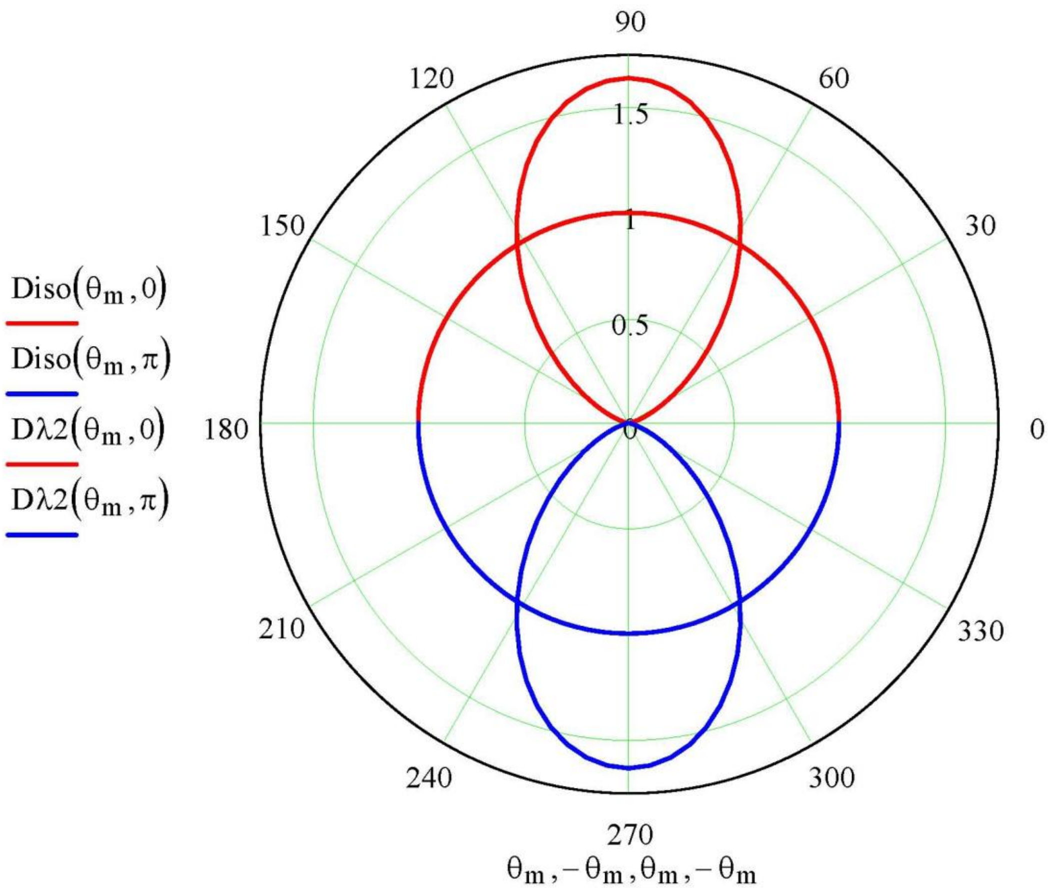
Directivity radiation patterns in spherical coordinates for isotropic and dipole antennas

$$\mathrm{Diso}(\theta, \phi) := 1$$

$$\mathrm{D\lambda 2}(\theta, \phi) := 1.64 \cdot \left| (\sin(\theta))^3 \right|$$

Polar 2D directivity radiation patterns in *x-y* plane for isotropic and dipole antennas

(Note: red traces for $\phi = 0$ and blue traces for $\phi = \pi$.)



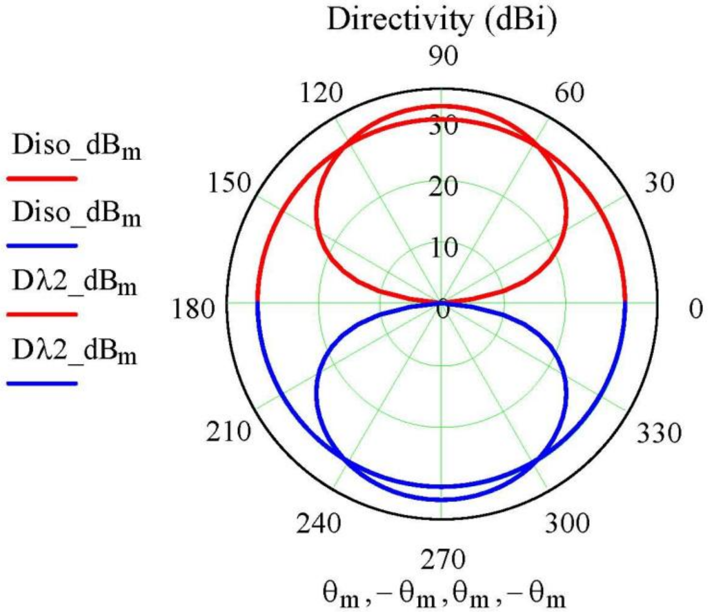
$\lambda/2$ Dipole directivity radiation patterns cont.

Polar 2D directivity radiation patterns (dBi) in *x-y* plane for isotropic & dipole antennas

$$\text{Diso\_dB}_m := \text{if}\left(\text{Diso}\left(\theta_m, 0\right) < 10^{-3}, 0, 30 + 10 \cdot \log\left(\text{Diso}\left(\theta_m, 0\right)\right)\right)$$

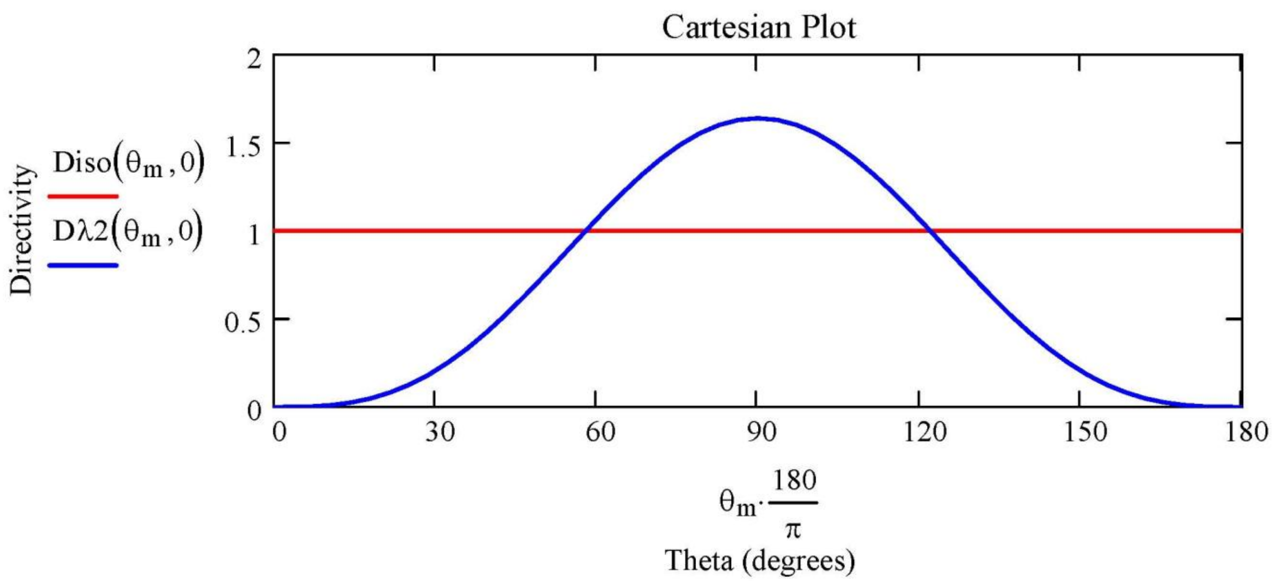
$$\text{D}\lambda\text{2\_dB}_m := \text{if}\left(\text{D}\lambda 2\left(\theta_m, 0\right) < 10^{-3}, 0, 30 + 10 \cdot \log\left(\text{D}\lambda 2\left(\theta_m, 0\right)\right)\right)$$

Must put in an offset (e.g., 30 dB) to work around a limitation in the MathCad polar plotting routine that center be >= 0.



Center of plot is really -30 dB

Cartesian 2D directivity radiation patterns in *x-y* plane for isotropic and dipole antennas



$\lambda/2$ Dipole directivity radiation patterns cont.

Project antenna patterns from spherical coordinates into cartesian for plotting purposes

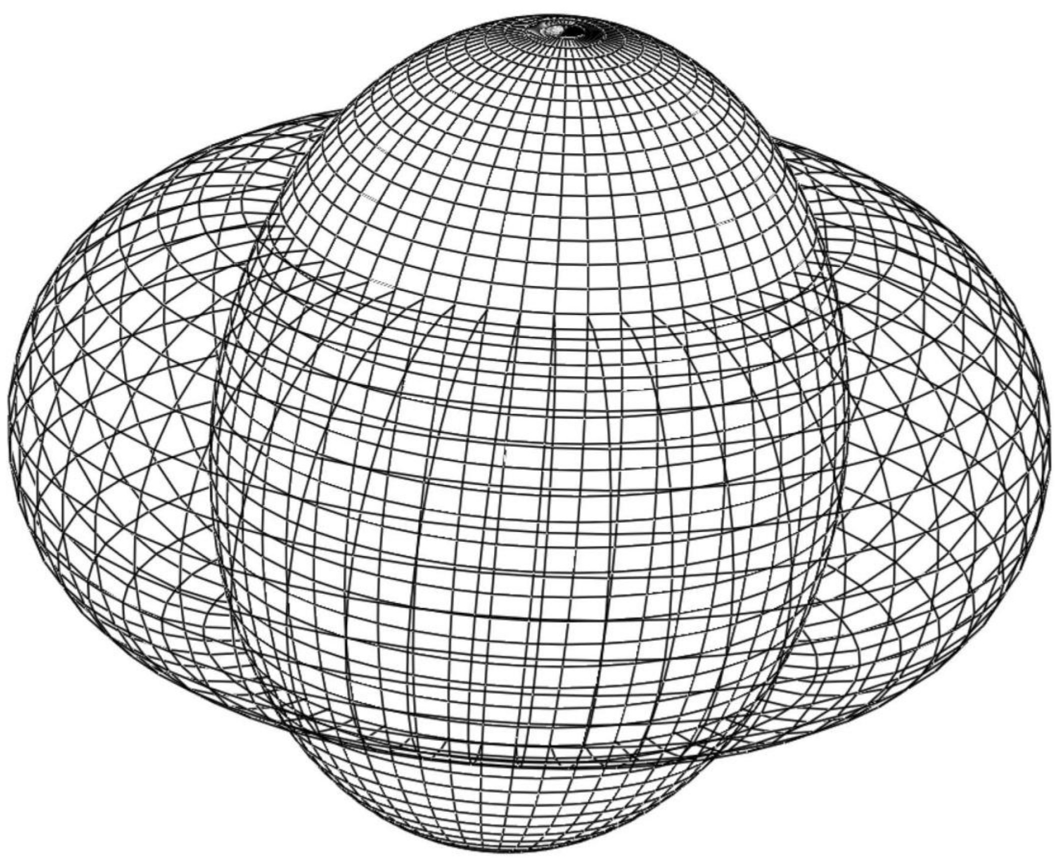
$X_{(m,n)} := \mathrm{Diso}(\theta_m, \phi_n) \cdot \sin(\theta_m) \cdot \cos(\phi_n)$ $\quad$ $Y_{(m,n)} := \mathrm{Diso}(\theta_m, \phi_n) \cdot \sin(\theta_m) \cdot \sin(\phi_n)$

$Z_{(m,n)} := \mathrm{Diso}(\theta_m, \phi_n) \cdot \cos(\theta_m)$

$X\lambda2_{(m,n)} := D\lambda2(\theta_m, \phi_n) \cdot \sin(\theta_m) \cdot \cos(\phi_n)$ $\quad$ $Y\lambda2_{(m,n)} := D\lambda2(\theta_m, \phi_n) \cdot \sin(\theta_m) \cdot \sin(\phi_n)$

$Z\lambda2_{(m,n)} := D\lambda2(\theta_m, \phi_n) \cdot \cos(\theta_m)$

3D Plot



$(X, Y, Z), (X\lambda2, Y\lambda2, Z\lambda2)$

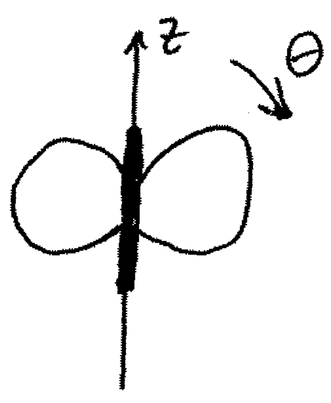
Principal Patterns - usually applies to linearly polarized antennas (tip of $\bar{E}$ traces out a line wrt time) and are usually broken into two orthogonal planes.

E-plane - antenna pattern for plane containing the electric field vector and direction of maximum radiation

H-plane - same w/ magnetic field vector

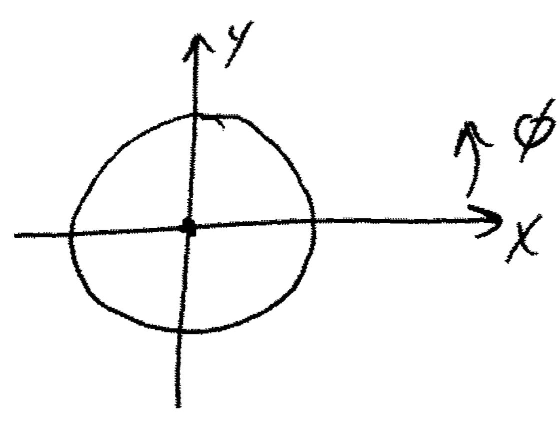
ex. Dipole Antena ($\bar{E} = \hat{a}_\theta E_\theta$, $\bar{H} = \hat{a}_\phi H_\phi$)

E-plane



@ some ϕ_0

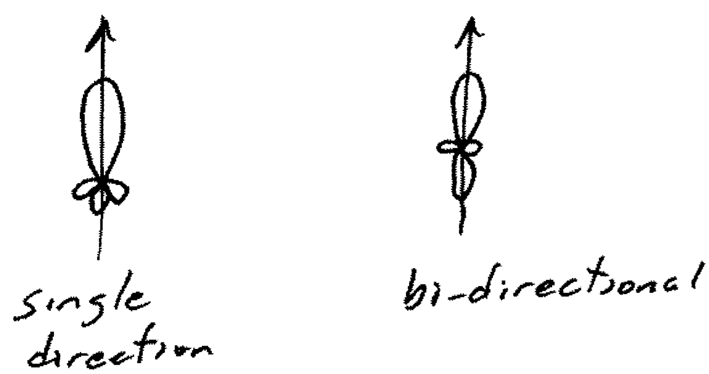
H-plane



Isotropic Radiator / Antenna - hypothetical, lossless antenna that radiates equally in all directions (spherical shell radiation pattern). It is used as a reference against which to compare real antennas. e.g., directivities / gains often expressed in dBi - decibels wrt isotropic.

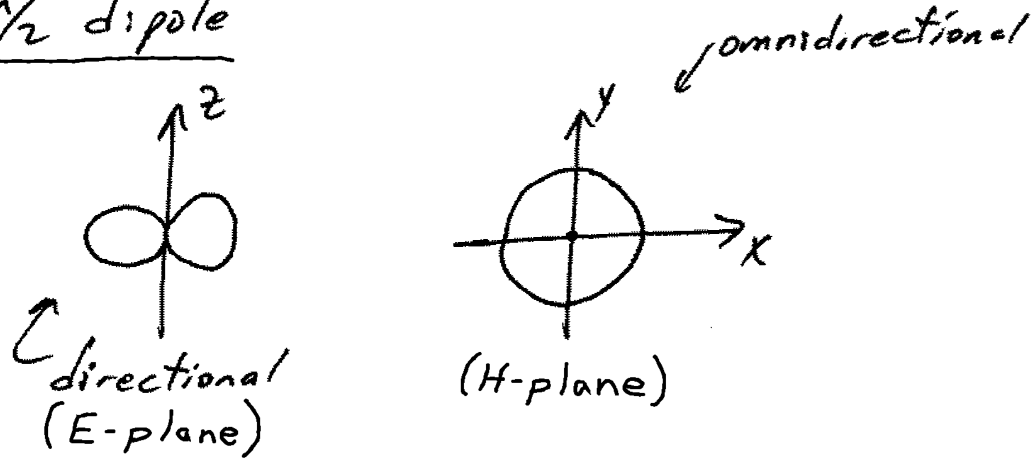
Directional Antennas - antennas having the property of radiating / receiving EM waves more effectively / preferentially in some directions than others.

ex. LPDA, Yagi-Uda, Horns

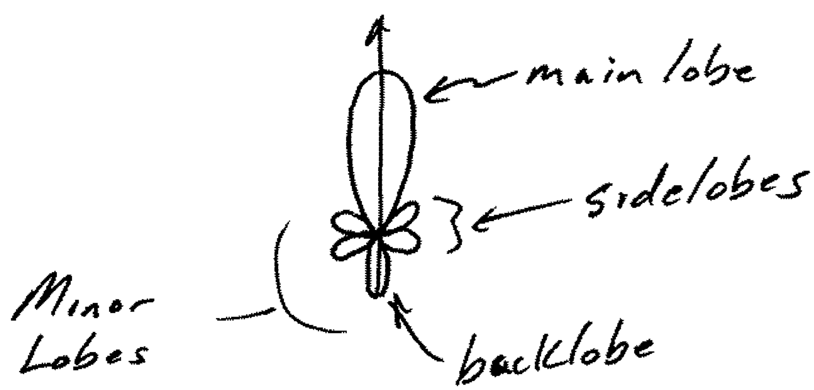


Omnidirectional Antennas - antennas having an essentially non-directional radiation pattern in a given plane and a directional pattern in any orthogonal plane.

ex. $\lambda/2$ dipole



Lobes – parts of radiation pattern bounded by areas of relatively weak radiation



Mainlobe – lobe containing direction of maximum radiation

Sidelobe – lobe(s) in direction(s) other than intended (usually all but mainlobe)

Minor Lobe – any lobe but mainlobe

Back lobe – lobe located ~180° from mainlobe.

Field Regions

- the space surrounding an antenna is usually divided into three (3) regions

Reactive near-field - portion of near-field closest to antenna where the reactive field components are dominant (largest)

Rule of thumb $\quad R < 0.62\sqrt{D^3/\lambda}$

where D is the largest dimension of the antenna.

Fresnel/radiating near-field - portion of near-field between the reactive near-field and the far-field region where radiated fields predominate and the angular field distrib. is dependent on distance from antenna.

$$0.62\sqrt{\frac{D^3}{\lambda}} < R < 2D^2/\lambda$$

Fraunhofer/far-field - region of space where the radiated fields predominate and where the angular field distribution is independent of distance from antenna. $\quad R > 2D^2/\lambda$

Radiated Power

Poynting Vector $\quad \bar{\mathcal{P}} = \bar{\mathcal{E}} \times \bar{\mathcal{H}} \quad \left(\frac{V}{m} \cdot \frac{A}{m} = W/m^2\right)$ ← Power density

Time-Ave Poynting Vector (Power Density vector) $\quad \bar{\mathcal{P}}_{ave} = \frac{1}{2}\,\mathrm{Re}\{\bar{E} \times \bar{H}^*\}$ ← phasor fields

$$= \frac{1}{T}\int_{t_o}^{t_o+T} \bar{\mathcal{P}}\, dt$$

TEM/F.F. region → $\left|\bar{\mathcal{P}}_{ave}\right| = \frac{1}{2}\,\eta\,|\bar{H}|^2 = \frac{|\bar{E}|^2}{2\eta} \quad (W/m^2)$

Note: $|\bar{E}|^2 = \bar{E} \cdot \bar{E}^*$ ← need both dot product + complex conjugate

The total power radiated by an antenna can be found by integrating the time-ave Poynting vector over a closed surface surrounding an antenna.

$$\boxed{P_{rad} = \oint\!\!\!\oint_S \bar{\mathcal{P}}_{ave} \cdot d\bar{s} \quad (W)}$$

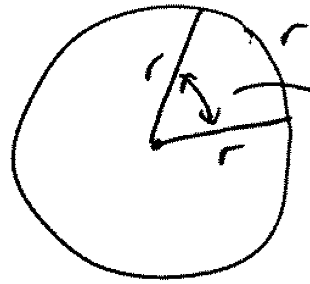
sph. coordinates $d\bar{s} = \hat{a}_r\, r^2 \sin\theta\, d\theta\, d\phi$

where $d\bar{s}$ is the differential surface area (points outward) and S is a closed surface around the antenna.

Radiation Intensity

First let's review how a radian and steradian (solid angle) are defined

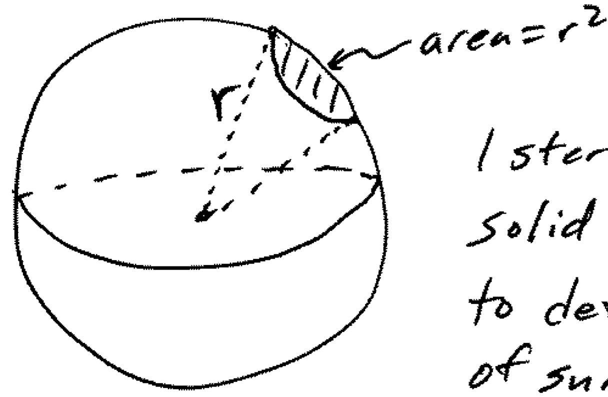
Radian



1 radian is the angle subtended on a circle when the arc length is equal to the radius r.

There are 2π radians in a complete circle.

Steradian



1 steradian is the solid angle subtended to define a patch of surface area on a sphere equal to the radius squared.

There are 4π sr in a sphere.

Radiation Intensity $U(\theta,\phi)$ – defined as the power <u>radiated</u> from an antenna per unit solid angle in a given direction (W/sr).

By its definition, the radiation intensity is related to the radiated power by

$$P_{rad} = \oint\!\!\oint_S U(\theta,\phi)\, d\Omega \quad \text{where } d\Omega = \sin\theta\, d\theta\, d\phi \text{ in spherical coordinates}$$

$$\boxed{P_{rad} = \oint\!\!\oint_S U(\theta,\phi) \sin\theta\, d\theta\, d\phi}$$

We can compare this expression w/ our previous P_{rad} equation to find how to calculate $U(\theta,\phi)$

$$P_{rad} = \oint\!\!\oint_S \bar{P}_{ave} \cdot d\bar{s} = \oint\!\!\oint_S U(\theta,\phi)\, d\Omega$$

$$\oint\!\!\oint_S P_{ave}\, ds = \oint\!\!\oint_S U(\theta,\phi)\, d\Omega$$

$$\oint\!\!\oint_S \frac{1}{2\eta} |\bar{E}|^2 r^2 \sin\theta\, d\theta\, d\phi = \oint\!\!\oint_S \tfrac{1}{2} \eta |\bar{H}|^2 r^2 \sin\theta\, d\theta\, d\phi$$

$$= \oint\!\!\oint_S U(\theta,\phi) \sin\theta\, d\theta\, d\phi$$

By comparison

$$\boxed{U(\theta,\phi) = r^2 |\bar{P}_{ave}| = \frac{r^2}{2\eta} |\bar{E}|^2 = \frac{r^2 \eta}{2} |\bar{H}|^2}$$

In the far-field, both $\bar{E}$ + $\bar{H}$ only have θ and/or ϕ components (for a TEM wave to propagate <u>away</u> from the antenna at the origin, it can't have radial $\bar{E}$ or $\bar{H}$ components)

Therefore-
$$|\bar{E}|^2 = |E_\theta|^2 + |E_\phi|^2$$
$$|\bar{H}|^2 = |H_\theta|^2 + |H_\phi|^2$$

Note: $|A|^2 = AA^*$ where A is a complex #.

The <u>average</u> radiation intensity of an antenna is

$$U_{ave} = \frac{P_{rad}}{4\pi} = \frac{\oint_S U\,d\Omega}{4\pi} \leftarrow \text{Total sr in a sphere}$$

For an isotropic radiator

$$\boxed{U_{iso}(\theta,\phi) = U_{ave} = U_0 = \frac{P_{rad}}{4\pi}}$$

→ $U(\theta,\phi)$ is a used to find/define the next important concepts - <u>directivity</u> + <u>gain</u>.

Directivity (AKA: Directive Gain)

Directivity – a measure of the concentration of the radiated power in a given direction

or

the ratio of the radiation intensity in a given direction to the radiation intensity average. (i.e. comparing the radiation intensity of an actual antenna to that of an isotropic radiator)

$$\boxed{D(\theta,\phi) = G_d(\theta,\phi) = \frac{U(\theta,\phi)}{U_{ave}} = \frac{U(\theta,\phi)}{P_{rad}/4\pi} = \frac{4\pi U(\theta,\phi)}{P_{rad}} \quad \text{(unitless)}}$$

If someone just says/refers to the "directivity" of an antenna, they mean the <u>maximum</u> directivity

$$\boxed{\left[G_d(\theta,\phi)\right]_{max} = D_o = \frac{U_{max}}{U_{ave}} = \frac{4\pi U_{max}}{P_{rad}}} \quad D_o > 1$$

$$D_{isotropic} = \frac{\cancel{U_{max}} \rightarrow U_{ave}}{U_{ave}} = 1 \leftarrow \text{smallest possible value}$$

ex. For small dipole, the far-zone fields are:

$$\bar{E} = \hat{a}_\theta \, jE_0 \, \frac{e^{-jkr}}{r} \sin\theta \quad (V/m)$$

$$\bar{H} = \hat{a}_\phi \, \frac{jE_0}{\eta} \, \frac{e^{-jkr}}{r} \sin\theta \quad (A/m)$$

$$\eta = \sqrt{\mu/\varepsilon}$$

The time-average Poynting vector is:

$$\bar{P}_{ave} = \tfrac{1}{2}\,\mathrm{Re}\{\bar{E}\times\bar{H}^*\} = \tfrac{1}{2}\,\mathrm{Re}\left\{\hat{a}_\theta \, jE_0 \frac{e^{-jkr}}{r}\sin\theta \times \hat{a}_\phi \frac{-jE_0^* e^{+jkr}\sin\theta}{\eta \, r}\right\}$$

$$= \tfrac{1}{2}\,\mathrm{Re}\left\{\hat{a}_r \frac{|E_0|^2}{\eta}\frac{(1)\sin^2\theta}{r^2}\right\} = \underline{\underline{\hat{a}_r \frac{|E_0|^2}{2\eta}\frac{\sin^2\theta}{r^2} \quad (W/m^2)}}$$

$= \bar{P}_{rad}$ ← radiated power density

$$P_{rad} = P_{ave} = \oint_S \bar{P}_{ave}\cdot d\bar{s} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \hat{a}_r \frac{|E_0|^2}{2\eta}\frac{\sin^2\theta}{r^2} \cdot \hat{a}_r r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{|E_0|^2}{2\eta}\int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin\theta \, d\theta = \frac{|E_0|^2}{2\eta}\,\phi\Big|_0^{2\pi}\left(-\cos\theta + \frac{\cos^3\theta}{3}\right)\Big|_0^{\pi}$$

$$= \frac{|E_0|^2}{2\eta}(2\pi - 0)\left[\left(-(-1) + \frac{(-1)^3}{3}\right) - \left(-1 + \frac{1^3}{3}\right)\right]$$

$$\underline{\underline{P_{rad} = \frac{4\pi |E_0|^2}{3\eta} \quad (W)}}$$

$$U(\theta,\phi) = U(\theta) = r^2 |\bar{P}_{ave}| = \underline{\underline{\frac{|E_0|^2}{2\eta}}}\sin^2\theta \quad (W/sr)$$

$$D(\theta,\phi) = D(\theta) = \frac{4\pi U(\theta)}{P_{rad}} = \underline{\underline{3/2 \, \sin^2\theta}}$$

$$\underline{\underline{D_0 = D_{max} = 3/2 \; (@\, \theta = \pi/2)}}$$

Directivity cont.

When not dimensionless, directivity is usually expressed in decibels relative to the directivity of an isotropic radiator

$$D(dB_i) = 10\log_{10}\left(\frac{D}{D_{iso}}\right) = 10\log_{10}\left(\frac{D}{1}\right)$$

ex. From previous example

$$D_0 = 3/2$$

$$D_0(dB_i) = 10\log_{10}\left(\frac{3/2}{1}\right) = \underline{\underline{1.761\,dB_i}}$$

ex. $\lambda/2$ dipole

$$D_{\lambda/2}(\theta,\phi) \simeq 1.64\,\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}$$

$$D_{0,\lambda/2} = 10\log_{10}\left(\frac{1.64}{1}\right) = \underline{\underline{2.15\,dB_i}}$$

→ A common form of radiation patterns are directivity plots.

Gain (AKA Power Gain)

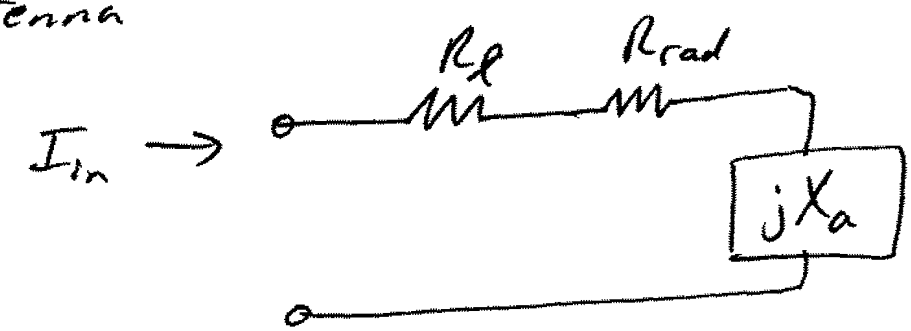
→ directivity is related to radiated power (as opposed to input power), therefore, it does not take into account antenna losses

$$\boxed{G_p(\theta,\phi) = G(\theta,\phi) = \frac{U(\theta,\phi)}{\left(P_{in}/4\pi\right)} = \frac{4\pi\, U(\theta,\phi)}{P_{in}}}$$

where $P_{in} = P_{losses} + P_{rad}$

$$= \tfrac{1}{2}\,|I_{in}|^2 R_\ell + \tfrac{1}{2}\,|I_{in}|^2 R_{rad}$$

per the circuit model for a transmitting antenna



Gain does not include "losses" due to impedance mismatch or polarization mismatches where some power available doesn't make it into the antenna.

Gain cont.

* Since $P_{in} \geq P_{rad}$, the gain will be less than or equal to the directivity.

* It is possible for the maximum gain G_o to be less than 1 (due to losses)

* If the word "gain" is used by itself, it (usually) implies the maximum gain G_o

$$\boxed{G_o = \left[G(\theta,\phi)\right]_{max} = \frac{4\pi U_{max}}{P_{in}}}$$

<u>Antenna efficiency</u> is defined as

$$\boxed{\eta_r = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_\ell} = \frac{G}{D} \leq 1}$$

$\hookrightarrow \boxed{G = \eta_r D}$ ☆

→ For many antennas, $\eta_r \simeq 1$. So, engineers often use the terms "directivity" and "gain" interchangeably.

Relative Gain

This is the ratio of the gain of an antenna in a given direction to the gain of a reference antenna (e.g. dipole, horn, ...)

Meas. → $G_{Rel}(\theta,\phi) = G(\theta,\phi)\, G_{ref}(\theta,\phi)$ ← dimensionless

In decibels ← usually maximum gain

$$G_{Rel}(\theta,\phi)\,(dB_{ref}) = G(\theta,\phi)\,(dBi) + G_{ref}(\theta,\phi)\,(dBi)$$

e.g. $G_{R,\lambda/2}(\theta,\phi) = G(\theta,\phi)\, G_{\lambda/2}$

$$G_{R,\lambda/2}(dBd) = G(\theta,\phi)\,(dBi) + G_{\lambda/2}\,(dBi)$$

$$12\ dBd = G(\theta,\phi)\,(dBi) + 2.15\ dBi$$

$$G(\theta,\phi)\,(dBi) = 12\ dBd - 2.15\ dBi = \underline{\underline{9.85\ dBi}}$$

Why? This is how radiation pattern measurements taken experimentally.

→ Most modern texts, articles, ... will convert relative gains to gain wrt an isotropic radiator (dBi).

Half-Power Beam Width or Beamwidth (HPBW)

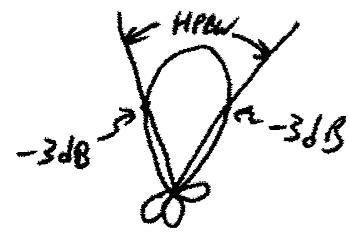
In a radiation pattern cut (2D pattern) containing the direction of the maximum of the main lobe, the HPBW is the angle between the two directions at which the power is one-half the maximum value.

* For $|E|$, half power points at $\frac{|E|_{max}}{\sqrt{2}} = 0.7071\,|E|_{max}$ since $P \propto |E|^2$
* For $U(\theta,\phi)$, half power points at $0.5\,U_{max}$.
* In decibels, the half power points are called the "3dB" points where the quantity in decibels is down 3dB or -3dB from the maximum

$$10\log_{10}\left(\frac{0.5\,U_{max}}{U_{max}}\right) = 10\log_{10}(0.5) = \underline{-3.0103\text{ dB}}$$

$$20\log_{10}\left(\frac{\frac{1}{\sqrt{2}}|E|_{max}}{|E|_{max}}\right) = 20\log_{10}\left(\tfrac{1}{\sqrt{2}}\right) = \underline{-3.0103\text{ dB}}$$

* HPBW is an important parameter for evaluating antennas and is closely related to directivity (narrow HPBW means larger D_0)



Bandwidth (BW)

Range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard.

e.g. input impedance, VSWR, radiation pattern

⇒ There are two ways of expressing bandwidth depending on whether an antenna is considered narrowband or wideband.

Narrowband – BW expressed as a fraction or percentage of the center frequency

$$\%\,BW = \frac{\overbrace{f_{high} - f_{low}}^{\leftarrow BW}}{f_{center}} \times 100\%$$

ex. UHF TV stations are allocated 6MHz/ch, So channel 14 470-476 MHz has a BW of

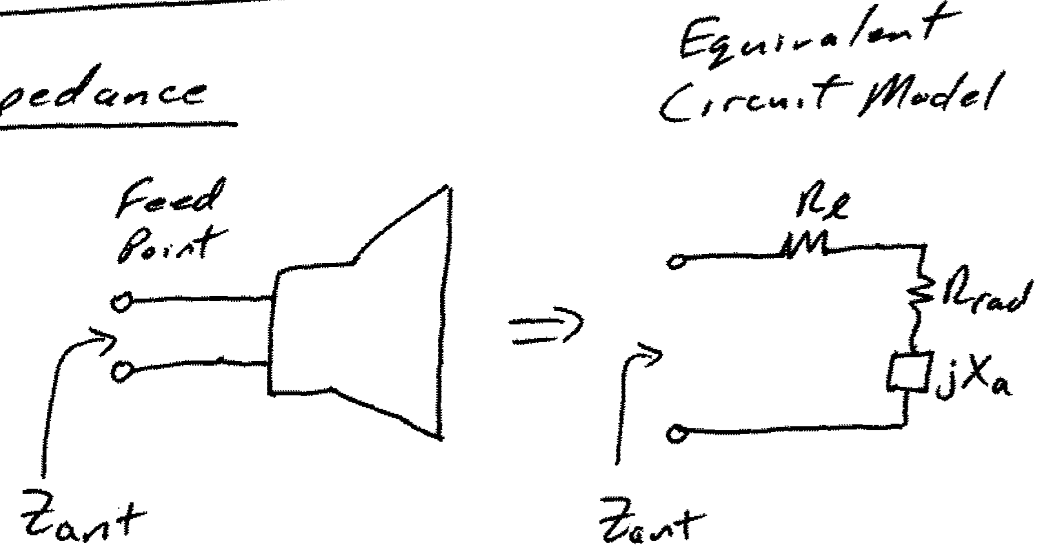
$$\frac{6\,MHz}{473\,MHz} \times 100\% = \underline{\underline{1.2766\%}}$$

Wideband – BW expressed as a ratio of the upper/higher frequency to lower frequency

$$BW = \frac{f_{up}}{f_{low}}$$ e.g. 10:1 means $f_{up} = 10\, f_{low}$

Antenna Characteristics

Input Impedance



$$Z_{ant} = R_a + jX_a \quad (\Omega)$$
$$= (R_{rad} + R_\ell) + jX_a$$

R_{rad} – radiation resistance (accounts for power 'lost' to radiation)

R_ℓ – loss resistance (accounts for power lost to various mechanisms not radiation, e.g. ohmic/conduction, dielectric, …)

X_a – antenna reactance (accounts for power stored in fields near antenna, i.e. non-propagating/reactive fields)

Input impedance cont.

→ Z_{ant} is very much a function of frequency

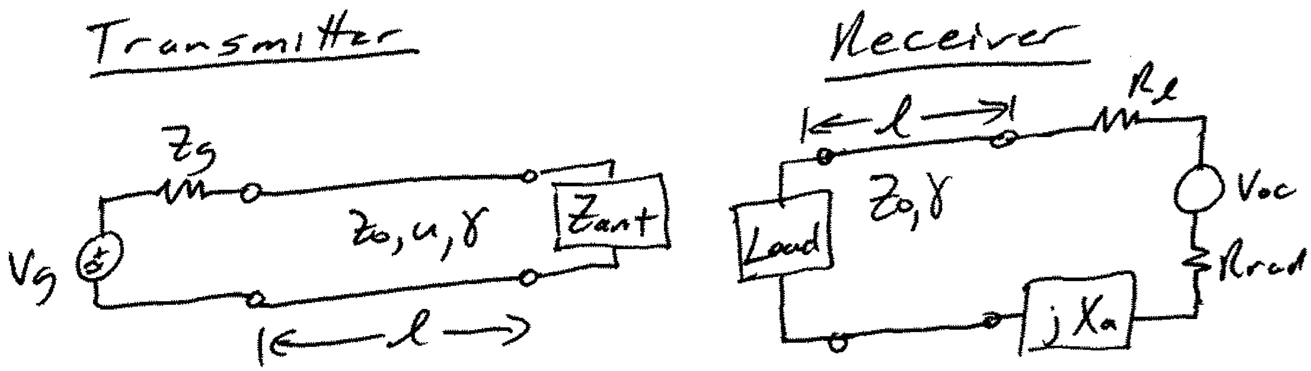
→ wideband antennas $Z_{ant} \approx$ constant over operating bandwidth

→ Z_{ant} key in determining

$$\Gamma_{ant} = \frac{Z_{ant} - Z_0}{Z_{ant} + Z_0}$$

$$VSWR = \frac{1 + |\Gamma_{ant}|}{1 - |\Gamma_{ant}|}$$

Typical Circuits



Ideally, everything is matched, R_ℓ is small, and max. power goes to R_{rad} or Load.

Effective Area

For a receiving antenna, the effective area is the ratio of the time-average power received (or delivered to the load) to the time-average power density (magnitude of time-average Poynting vector) of the incident EM wave at the antenna.

$$\boxed{A_e = \frac{P_r(W)}{\mathcal{P}_{ave}(W/m^2)} \quad (m^2)}$$

very much a function of angle

→ This is the area that would be required to capture the power received from the power density at this point

$$P_r = \int_S \bar{\mathcal{P}}_{ave} \cdot d\bar{s} = \mathcal{P}_{ave} A_e$$

→ Good measure of an antennas ability to extract power from an incident EM wave

→ The effective area, as one might expect, is related to the directivity (lossless) or gain (lossy) of an antenna.

Effective Area cont.

For lossless antennas

$$\boxed{A_e(\theta,\phi) = \left(\frac{\lambda^2}{4\pi}\right) D(\theta,\phi)}$$

For lossy antennas

$$\boxed{A_e(\theta,\phi) = \left(\frac{\lambda^2}{4\pi}\right) \eta_r D(\theta,\phi) = \left(\frac{\lambda^2}{4\pi}\right) G(\theta,\phi)}$$

* Most often, if someone refers to the "effective area" of an antenna, they mean the <u>maximum</u> effective area

$$\boxed{A_{em} = \left(\frac{\lambda^2}{4\pi}\right) D_0 \quad \text{or} \quad \left(\frac{\lambda^2}{4\pi}\right) G_0}$$

ex. For a $\lambda/2$ dipole, $D_0 = 1.64$ and $\eta_r = 0.99$ at a frequency of 490 MHz, find A_{em}.

$$\lambda = u/f = \frac{2.998\times10^8}{490\times10^6} = 0.611837\,m$$

$$A_{em} = \left(\frac{0.611837^2}{4\pi}\right) 0.99(1.64) = \underline{\underline{0.048366\ m^2}}$$

Effective Area cont.

ex. If a TEM wave where $|\bar{E}| = 2$ V/m is incident at broadside on the $\lambda/2$ dipole of the previous example, what is the maximum power received?

$$\rightarrow A_{em} = 0.04885\, m^2 = \frac{P_r}{\mathcal{P}_{ave}} \qquad \mathcal{P}_{ave} = \frac{|\bar{E}|^2}{2\eta}$$

$$P_r = A_{em}\,\mathcal{P}_{ave} = (0.048366)\,\frac{2^2}{2(376.73)}$$

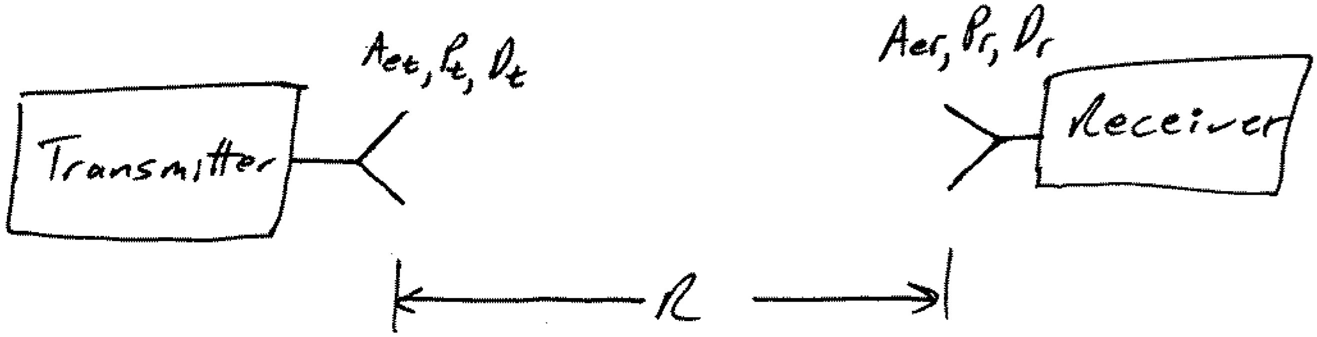
$$\underline{P_r = 0.257\, mW}$$

only achieved if antenna matched w/ receiver/load and the incident EM wave is polarization matched with the antenna.

$\rightarrow$ For wire antennas (dipoles, loops, ...), it is easily possible that $A_{em} > A_{phys}$ the physical area of the antenna. For aperture antennas (horns, waveguides, dishes, ...), the max. effective area usually is 20% to 80% of the physical area.

Friis Transmission Equation

– This equation relates the power transmitted (radiated) by one antenna to what is received by another antenna.



$$D_t(\theta,\phi) = \frac{4\pi U_t(\theta,\phi)}{P_t} = \frac{4\pi R^2 \mathcal{P}_{ave}(\theta,\phi)}{P_t} \quad \text{@ Receiver}$$

Solving for the average power density at the receiver

$$\mathcal{P}_{ave} = \frac{P_t}{4\pi R^2} D_t(\theta,\phi)$$

The maximum possible power received can be found using the effective area as

$$P_r = \mathcal{P}_{ave} A_{er} = \mathcal{P}_{ave}\left(\frac{\lambda^2}{4\pi}\right) D_r(\theta,\phi)$$

Now, we can combine expressions to get the Friis Transmission Eqn.

Friis Trans. Eqn. cont.

Simplest form

$$\boxed{P_r = D_t(\theta,\phi)\, D_r(\theta,\phi) \left[\frac{\lambda}{4\pi R}\right]^2 P_t}$$

This equation can be put in decibel form as

$$\boxed{\underbrace{P_r\,(dBx)}_{\substack{dBm\\ \text{or}\\ dBW}} = D_t\,(dBi) + D_r\,(dBi) + \underbrace{20\log_{10}\left(\frac{\lambda}{4\pi R}\right)}_{\substack{\text{space}\\ \text{loss}}} + \underbrace{P_t\,(dBx)}_{\substack{dBm\\ \text{or}\\ dBW}}}$$

Other factors considered in more complete forms of the Friis Transmission Equation are antenna efficiencies (or replace directivities w/ gains), impedance mismatches, + polarization mismatches.

$$\frac{P_r}{P_t} = \underbrace{\left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)}_{\substack{\text{impedance}\\ \text{matching}}} \underbrace{\left(\frac{\lambda}{4\pi R}\right)^2}_{\substack{\text{space}\\ \text{loss}}} \underbrace{\eta_t \overbrace{D_t(\theta,\phi)}^{G_t(\theta,\phi)}\, \eta_r \overbrace{D_r(\theta,\phi)}^{G_r(\theta,\phi)}}_{\substack{\text{antenna}\\ \text{efficiencies}}} \underbrace{\left|\hat{\rho}_t \cdot \hat{\rho}_r\right|^2}_{\substack{\text{Polarization}\\ \text{mis-match}}}$$

ex. Friis Trans. Eqn.

At 2.4 GHz, a horn antenna transmits 100 W of power to another antenna located 5 Km away. What power is received if the transmitting antenna has a directivity of 20 dBi and the receiving antenna has a directivity of 15 dBi?

$$\lambda = c/f = \frac{2.998\times10^{8}\,\text{m/s}}{2.4\times10^{9}\,\text{Hz}} = 0.12491\bar{6}\ \text{m}$$

__Method 1__

$$P_t(\text{dBm}) = 10\log_{10}\left(\frac{100}{10^{-3}}\right) = 50\ \text{dBm}$$

Space loss $20\log_{10}\left(\frac{\lambda}{4\pi R}\right) = 20\log_{10}\left(\frac{0.12491\bar{6}}{4\pi\,5000}\right) = -114.03\ \text{dB}$

$$P_r(\text{dBm}) = D_r(\text{dBi}) + D_t(\text{dBi}) + 20\log_{10}\left(\frac{\lambda}{4\pi R}\right) + P_t(\text{dBm})$$

$$= 15 + 20 - 114.03119 + 50$$

$$\underline{P_r(\text{dBm}) = -29.03119\ \text{dBm}}$$

$$P_r = (1\,\text{mW})\,10^{-29.03119/10} = \underline{1.24992\ \mu\text{W}}$$

same

__Method 2__

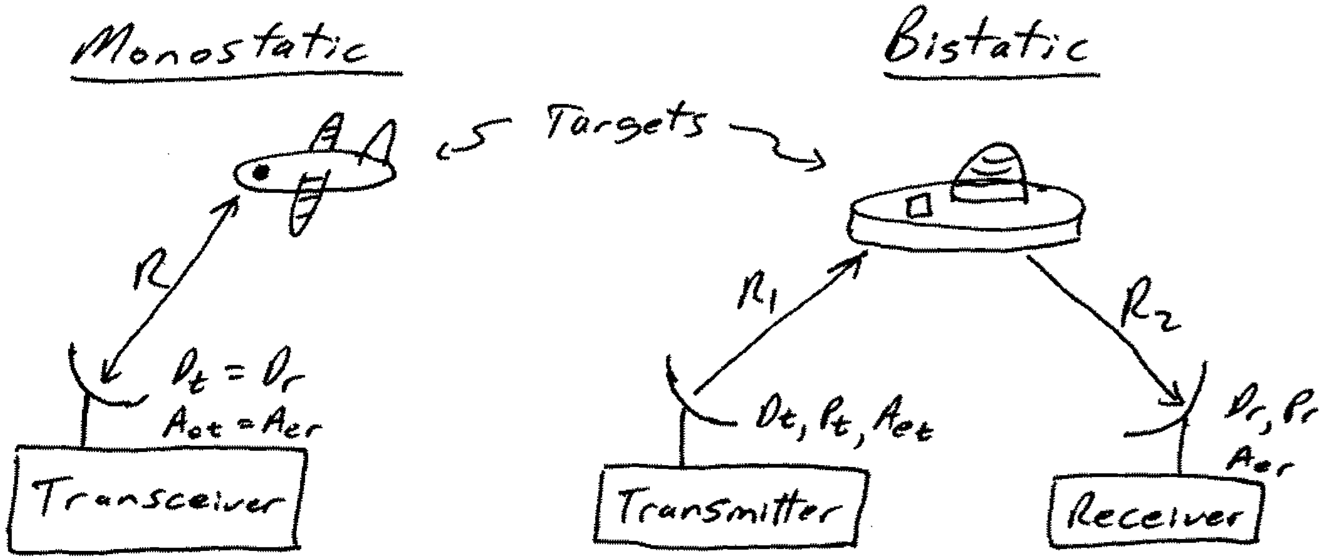
$D_r = 10^{15/10}$, $D_t = 10^{20/10}$ ← make unitless

$$P_r = 10^{1.5}\,10^{2}\left[\frac{0.12491\bar{6}}{4\pi\,5000}\right]^2 100 = \underline{1.24992\ \mu\text{W}}$$

$$P_r(\text{dBm}) = 10\log_{10}\left(\frac{1.24992\times10^{-6}}{10^{-3}}\right) = \underline{-29.03119\ \text{dBm}}$$

RADAR (Range) Equation

The problems considered by this equation are shown below.



To analyze this problem, the reflective/scattering properties of the target(s) need to be characterized.

Scattering Cross Section/RCS (σ)

Equivalent area intercepting that amount of power that, when scattered isotropically, produces at the radar a power density which is equal to that scattered (reflected) by the actual target.

$$P_{scattered} = \lim_{r \to \infty} \left[\frac{\sigma P_{incident}}{4\pi r^2} \right]$$

RADAR cont.

which leads to

$$\sigma = \lim_{R \to \infty} \left(4\pi R^2 \frac{\mathcal{P}_{scattered}}{\mathcal{P}_{inc}} \right)$$ ← units of m^2 (or dBsm)

when expressed in decibels

$$\sigma(dBsm) = 10 \log_{10}\left(\frac{\sigma}{1m^2}\right)$$

Again, we'll calculate the power density produced by the transmitter, this time at the target

$$\mathcal{P}_{inc} = \mathcal{P}_{ave} = \frac{D_t P_r}{4\pi R^2} \quad \text{or} \quad \frac{D_t P_r}{4\pi R_1^2}$$

↑ monostatic ↑ bistatic

The scattered power density (produced by target) at the receiver can be found using the scattering cross section as

$$\mathcal{P}_{scatt} = \frac{\sigma \mathcal{P}_{inc}}{4\pi R^2} \quad \text{or} \quad \frac{\sigma \mathcal{P}_{inc}}{4\pi R_2^2}$$

↑ monostatic case ↑ bistatic case

Table 2.2 RCS OF SOME TYPICAL TARGETS

Object	Typical RCSs [22] RCS (m^2)	RCS (dBsm)
Pickup truck	200	23
Automobile	100	20
Jumbo jet airliner	100	20
Large bomber *or* commercial jet	40	16
Cabin cruiser boat	10	10
Large fighter aircraft	6	7.78
Small fighter aircraft *or* four-passenger jet	2	3
Adult male	1	0
Conventional winged missile	0.5	−3
Bird	0.01	−20
Insect	0.00001	−50
Advanced tactical fighter	0.000001	−60

From Balanis, Antenna Theory (2nd Edition)

→ 10 GHz

→ maximum values (RCS is a function of frequency, angle, polarization, ...)

RADAR cont.

Combining these equations leads to:

Monostatic

or P_{rad}

$$P_r = \frac{\lambda^2 D^2 \sigma P_t}{(4\pi)^3 R^4}$$

$$= \frac{D^2}{4\pi}\left[\frac{\lambda}{4\pi R^2}\right]^2 \sigma P_t$$

Bistatic

$$P_r = \frac{D_t D_r}{4\pi}\left[\frac{\lambda}{4\pi R_1 R_2}\right]^2 \sigma P_t$$

Note: For monostatic radars: $R_1 = R_2 = R$

$D_t = D_r = D$

$\eta_t = \eta_r = \eta$

$\vdots$

A more complete version of the RADAR Eqn is

Bistatic

$$\frac{P_r}{P_t} = \underbrace{(1-|\Gamma_t|^2)(1-|\Gamma_r|^2)}_{\text{impedance mistmatches}}\underbrace{\left[\frac{\lambda}{4\pi R_1 R_2}\right]^2}_{\text{space loss}}\sigma \underbrace{\frac{\eta_t D_t \eta_r D_r}{4\pi}}_{\text{gains}} \underbrace{|\hat{\rho}_s \cdot \hat{\rho}_r|^2}_{\text{polarization mismatches}}$$

To boost power received:

- → make P_t large (1MW pulses not uncommon)
- → use high gain antennas (e.g. dishes, horns, arrays)
- → low frequencies (long λ)

ex. RADAR Range Eqn

An S-Band RADAR is being designed to detect an aircraft ($\sigma = 100\,m^2$) at a range of 100 miles. If the antenna (parabolic dish) has a directivity of 50 dBi at 3 GHz, how much power must be transmitted to receive at least 2 nW?

$$\lambda = u/f = \frac{2.998 \times 10^8 \ m/s}{3 \times 10^9 \ Hz} = 0.0999\bar{3}\ m$$

$$D = 10^{50/10} = 10^5$$

$$R = 100 \text{ miles} = 160.9\ Km$$

$$P_r = \frac{D^2}{4\pi}\left[\frac{\lambda}{4\pi R^2}\right]^2 \sigma P_t$$

$$2 \times 10^{-9} = \frac{(10^5)^2}{4\pi}\left[\frac{0.0999\bar{3}}{4\pi(160.9 \times 10^3)^2}\right]^2 100\ P_t$$

$$2 \times 10^{-9} = 7.5087 \times 10^{-15} P_t$$

$$\underline{\underline{P_t = 266.356\ KW = 0.266\ MW}}$$

This could be the peak value of a pulse.