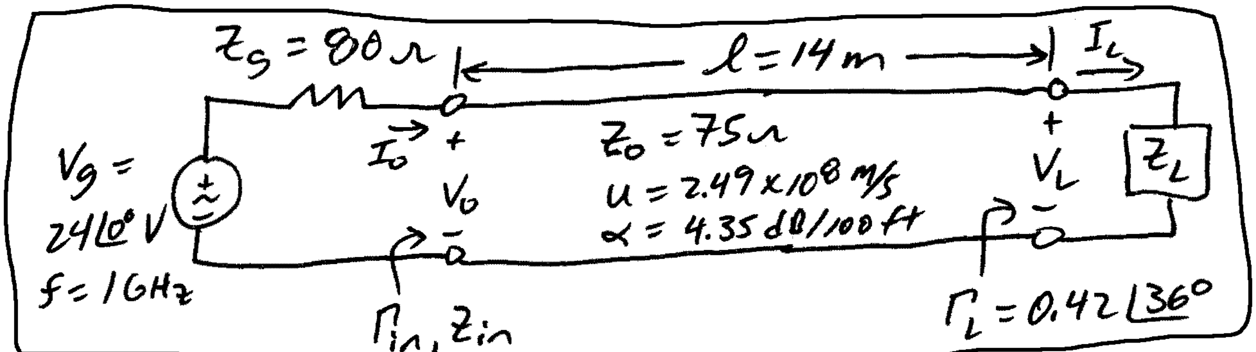
An RG-11 coaxial transmission line ($Z_0 = 75\ \Omega$, $u = 2.49 \times 10^8$ m/s, $\alpha = 4.35$ dB/100 ft) of length 14 m is terminated with a load. Using a vector network analyzer (VNA), a load reflection coefficient of $\Gamma_L = 0.42\angle 36°$ is measured. The transmission line is connected to a generator with $V_g = 24\angle 0°$ V and $Z_g = 80\ \Omega$ operating at 1 GHz. First, sketch the transmission line circuit. Then, determine (a) the attenuation (Np/m), phase (rad/m), & propagation constants, (b) SWR & load impedance, (c) input reflection coefficient & impedance, (d) phasor input current & voltage, (e) phasor forward voltage wave amplitude, (f) phasor load current & voltage, and (g) time-average power input and delivered to load.



a) $\alpha = \dfrac{4.35\ dB}{100\ ft}\left(\dfrac{3.28084\ ft}{1\ m}\right)\left(\dfrac{1\ Np}{20\log e\ dB}\right) \Rightarrow \underline{\underline{\alpha = 0.016431\ \frac{Np}{m}}}$

(11.14) $\beta = \dfrac{\omega}{u} = \dfrac{2\pi(1\times10^9)}{2.49\times10^8} \Rightarrow \underline{\underline{\beta = 25.2337\ \frac{rad}{m}}}$

(11.11) $\gamma = \alpha + j\beta \Rightarrow \underline{\underline{\gamma = 0.016431 + j\,25.2337\ m^{-1}}}$

b) (11.38a) $SWR = \dfrac{1+|\Gamma_L|}{1-|\Gamma_L|} = \dfrac{1+0.42}{1-0.42} \Rightarrow \underline{\underline{SWR = 2.448}}$

$Z_L = Z_0\dfrac{1+\Gamma_L}{1-\Gamma_L} = 75\,\dfrac{1+0.42\angle36°}{1-0.42\angle36°} \Rightarrow \underline{\underline{Z_L = 124.33 + j74.53\ \Omega}}$

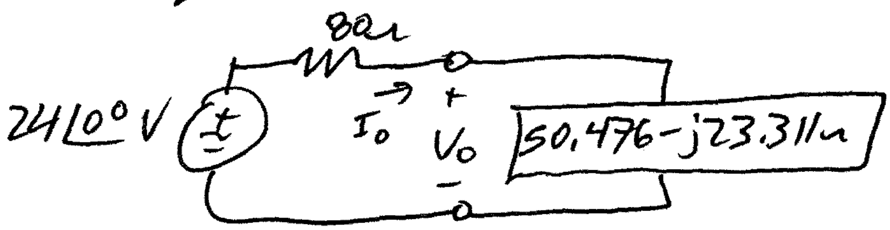
c) (11.37) $\Gamma_{in} = \Gamma_L e^{-2\gamma\ell} = (0.42\angle36°)\,e^{-2(0.0164+j25.234)14}$

$\underline{\underline{\Gamma_{in} = 0.2651\angle{-125.928°}}}$

c) cont. $Z_{in} = Z_0 \dfrac{1+\Gamma_{in}}{1-\Gamma_{in}} = 75 \dfrac{1+0.2651\angle{-125.93^\circ}}{1-0.2651\angle{-125.93^\circ}}$

$$\underline{\underline{Z_{in} = 50.476 - j23.311\,\Omega}}$$

d) Use equivalent circuit



$I_0 = \dfrac{24\angle 0^\circ}{80+(50.476-j23.311)} \Rightarrow \underline{\underline{I_0 = 0.1811\angle{10.13^\circ}\ A}}$

$V_0 = 24\angle 0^\circ \dfrac{50.476-j23.311}{80+(50.476-j23.311)} \Rightarrow \underline{\underline{V_0 = 10.0675\angle{-14.659^\circ}\ V}}$

e) (11.27a) $V_0^+ = \frac{1}{2}\left[V_0 + I_0 Z_0\right] = \frac{1}{2}\left[10.07\angle{-14.66^\circ} + 0.1811\angle{10.13^\circ}(75)\right]$

$$\underline{\underline{V_0^+ = 11.5546\angle{-0.3948^\circ}\ V}}$$

f) Notes $V_L = V_0^+ e^{-\gamma \ell}(1+\Gamma_L)$

$= 11.555\angle{-0.395^\circ}\, e^{-(0.0164+j25.234)14}\,(1+0.42\angle{36^\circ})$

$$\underline{\underline{V_L = 12.5066\angle{-70.918^\circ}\ V}}$$

$I_L = V_L / Z_L = \dfrac{12.5066\angle{-70.918^\circ}}{124.33+j74.53} \Rightarrow \underline{\underline{I_L = 0.08628\angle{-101.861^\circ}\ A}}$

g) $P_{in} = 0.5\,\mathrm{Re}\{V_0 I_0^*\} = 0.5\,\mathrm{Re}\{10.07\angle{-14.66^\circ}(0.1811\angle{-10.13^\circ})\} \Rightarrow \underline{\underline{P_{in} = 0.8275\,W}}$

$P_L = 0.5\,\mathrm{Re}\{V_L I_L^*\} = 0.5\,\mathrm{Re}\{12.51\angle{-70.92^\circ}(0.086\angle{101.9^\circ})\} \Rightarrow \underline{\underline{P_L = 0.4627\,W}}$