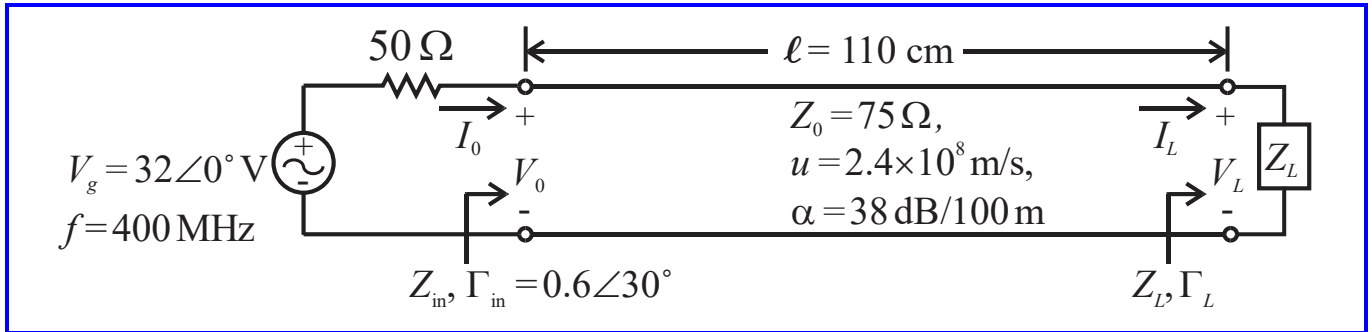


A lossy transmission line ($Z_0 = 75 \Omega$, $u = 2.4 \times 10^8$ m/s, $\alpha = 38$ dB/100 m) of length $\ell = 110$ cm is terminated with an unknown load Z_L . An input reflection coefficient $\Gamma_{in} = 0.6 \angle 30^\circ$ is measured. The transmission line is connected to a generator operating at 400 MHz with voltage $V_g = 32 \angle 0^\circ$ V and 50Ω impedance. First, sketch the transmission line circuit. Then, determine a) the propagation constant (1/m) & wavelength λ , b) input impedance as well as phasor current & voltage, c) load reflection coefficient & impedance and SWR, d) phasor forward voltage wave amplitude, e) phasor load current & voltage, and f) time-average power input and delivered to load.



a) $\alpha = 38 \text{ dB/100 m} = 0.38 \text{ dB/m} (1 \text{ Np} / 8.686 \text{ dB}) = 0.043749 \text{ Np/m}$

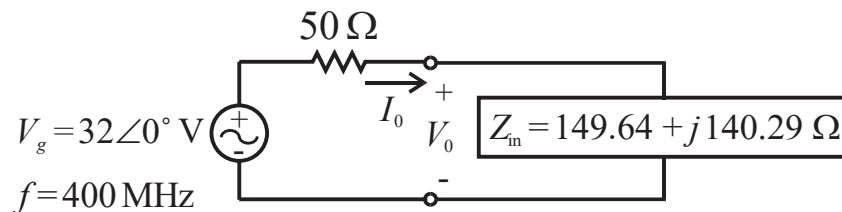
(11.21) $\beta = \omega/u = 2\pi(400 \times 10^6) / 2.4 \times 10^8 = 10.41976 \text{ Np/m}$

(11.11) $\gamma = \alpha + j\beta \Rightarrow \underline{\gamma = 0.043749 + j10.41976 \text{ (1/m)}}$

(11.14) $\lambda = u/f = 2.4 \times 10^8 / 400 \times 10^6 \Rightarrow \underline{\lambda = 0.6 \text{ m} = 60 \text{ cm}}$

b) Per notes, $Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 75 \left(\frac{1 + 0.6 \angle 30^\circ}{1 - 0.6 \angle 30^\circ} \right) \Rightarrow \underline{Z_{in} = 149.64 + j140.29 \Omega}$.

Draw equivalent circuit. Then, use voltage division and Ohm's Law.



$$V_0 = V_g \frac{Z_{in}}{Z_g + Z_{in}} = (32 \angle 0^\circ) \frac{149.6 + j140.3}{50 + (149.6 + j140.3)} \Rightarrow \underline{V_0 = 26.9 \angle 8.057^\circ \text{ (V)}}$$

$$I_0 = \frac{V_g}{Z_g + Z_{in}} = \frac{32 \angle 0^\circ}{50 + (149.6 + j140.3)} \Rightarrow \underline{I_0 = 0.1311 \angle -35.096^\circ \text{ (A)}}$$

c) Per notes, $\Gamma_L = \Gamma_{in} e^{2\gamma\ell} = (0.6\angle 30^\circ) e^{2(0.04375 + j10.472)1.1} \Rightarrow \underline{\Gamma_L = 0.6606\angle -90^\circ}.$

Per notes, $Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L} = 75 \left(\frac{1 + 0.6606\angle -90^\circ}{1 - 0.6606\angle -90^\circ} \right) \Rightarrow \underline{Z_L = 29.426 - j68.986 \Omega}.$

(11.38a) $SWR = S = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 0.6606}{1 - 0.6606} \Rightarrow \underline{S = SWR = 4.893}.$

d) Per (11.27), $V_0^+ = 0.5(V_0 + Z_0 I_0) = 0.5[(26.9\angle 8.06^\circ) + 75(0.1311\angle -35.1^\circ)]$
 $\Rightarrow \underline{V_0^+ = 17.367\angle -3.111^\circ \text{ (V)}}.$

e) Per notes,

$$V_L = V_0^+ e^{-\gamma\ell} (1 + \Gamma_L) = (17.367\angle -3.111^\circ) e^{-(0.04375 + j10.472)1.1} (1 + 0.6606\angle -90^\circ)$$

$$\Rightarrow \underline{V_L = 19.836\angle 23.439^\circ \text{ (V)}}.$$

$$I_L = \frac{V_L}{Z_L} = \frac{19.836\angle 23.439^\circ}{29.426 - j68.986} \Rightarrow \underline{I_L = 0.2645\angle 90.338^\circ \text{ (A)}}.$$

f) From Circuits,

$$P_{in} = 0.5 \operatorname{Re}(V_0 I_0^*) = 0.5 \operatorname{Re}[(26.9\angle 8.057^\circ)(0.1311\angle 35.096^\circ)]$$

$$\Rightarrow \underline{P_{in} = 1.2869 \text{ W}}.$$

$$P_L = 0.5 \operatorname{Re}(V_L I_L^*) = 0.5 \operatorname{Re}[(19.836\angle 23.439^\circ)(0.2645\angle -90.338^\circ)]$$

$$\Rightarrow \underline{P_L = 1.0292 \text{ W}} < P_{in} \text{ as expected}.$$