Repeat the previous problem if the transmission line is now assumed to be lossy with a measured attenuation of 18 dB/100 ft.

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A lossless transmission line (Z_0 = 50 Ω, $u = 2.08 \times 10^8$ m/s) of length 28.8λ is terminated with an unknown load. Using a vector network analyzer (VNA), an input reflection coefficient of $\Gamma_{in} = 0.50\angle 60°$ is measured. The transmission line (TL) is then connected to a generator with a voltage 36∠0° V and impedance 50 Ω operating at 2 GHz. Draw the TL circuit. Then, determine the (a) propagation constant & wavelength, (b) input impedance, (c) phasor current & voltage and time-average power at the input, (d) phasor forward voltage wave amplitude, (e) phasor current & voltage and time-average power at the load.

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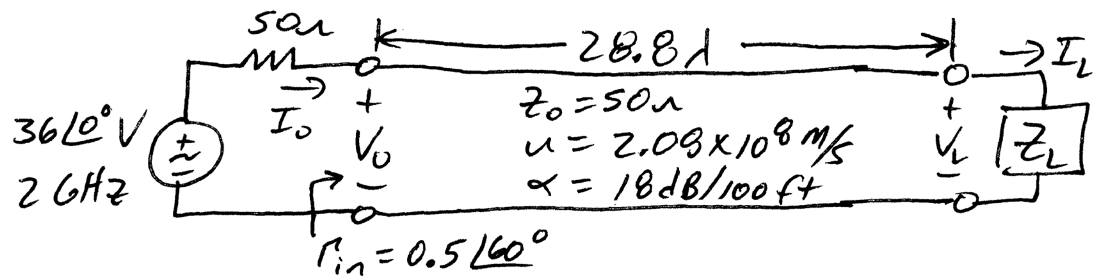

a) $\alpha = \frac{18\,dB}{100ft}\left(\frac{0.3048\,ft}{m}\right)\left(\frac{1\,Np}{20\log e\;dB}\right) \Rightarrow \alpha = 0.006316\ Np/m$

(11.21) $\beta = \omega/u = \frac{2\pi(2\times10^9)}{2.08\times10^8} \Rightarrow \beta = 60.4152\ rad/m$

(11.11) $\gamma = \alpha + j\beta = 0.006316 + j60.4152\ m^{-1}$

(11.14) $\lambda = 2\pi/\beta = \frac{2\pi}{60.4152} \Rightarrow \lambda = 0.104\ m$ (same)

b) Notes $Z_{in} = Z_0\frac{1+\Gamma_{in}}{1-\Gamma_{in}} = 50\frac{1+0.5\angle 60°}{1-0.5\angle 60°}$

$Z_{in} = 50 + j57.735\ \Omega$ (same)

c) Using Circuits

$I_0 = \frac{36\angle 0°}{50+(50+j57.735)} \Rightarrow I_0 = 0.3118\angle -30°\ A$

$V_0 = 36\angle 0°\ \frac{50+j57.735}{50+50+j57.735} \Rightarrow V_0 = 23.8118\angle 19.107°\ V$

c) cont. $P_{in} = \frac{1}{2} \mathrm{Re}\{V_0 I_0^*\} = \frac{1}{2} \mathrm{Re}\{23.8\angle 19.1^\circ (0.312\angle +30^\circ)\}$

$$\underline{\underline{P_{in} = 2.43 \text{ W}}} \quad \text{(same)}$$

d) (11.27) $V_0^+ = \frac{1}{2}[V_0 + I_0 Z_0] = \frac{1}{2}[23.8\angle 19.1^\circ + (0.312\angle -30^\circ)50]$

$$\underline{\underline{V_0^+ = 18\angle 0^\circ \text{ V}}} \quad \text{(same)}$$

e) Notes $\Gamma_L = \Gamma_{in} e^{2\gamma\ell} = 0.5\angle 60^\circ e^{2(0.00632+j60.415)28.8(0.104)}$

$$\Gamma_L = 0.5193\angle -84^\circ \quad \text{(bigger)}$$

$$I_L = \frac{V_0^+}{Z_0} e^{-\gamma\ell}(1-\Gamma_L)$$

$$= \frac{18\angle 0^\circ}{50} e^{-(0.00632+j60.415)28.8(0.104)}(1-0.5193\angle -84^\circ)$$

$$\underline{\underline{I_L = 0.3806\angle 100.638^\circ \text{ A}}}$$

$$V_L = V_0^+ e^{-\gamma\ell}(1+\Gamma_L)$$

$$= 18\angle 0^\circ e^{-(0.00632+j60.415)28.8(0.104)}(1+0.5193\angle -84^\circ)$$

$$\underline{\underline{V_L = 20.7355\angle 45.902^\circ \text{ V}}}$$

$P_L = \frac{1}{2} \mathrm{Re}\{V_L I_L^*\} = \frac{1}{2} \mathrm{Re}\{20.74\angle 45.9^\circ (0.381\angle -100.6^\circ)\}$

$$\underline{\underline{P_L = 2.2785 \text{ W}}} \quad \text{(Smaller)}$$