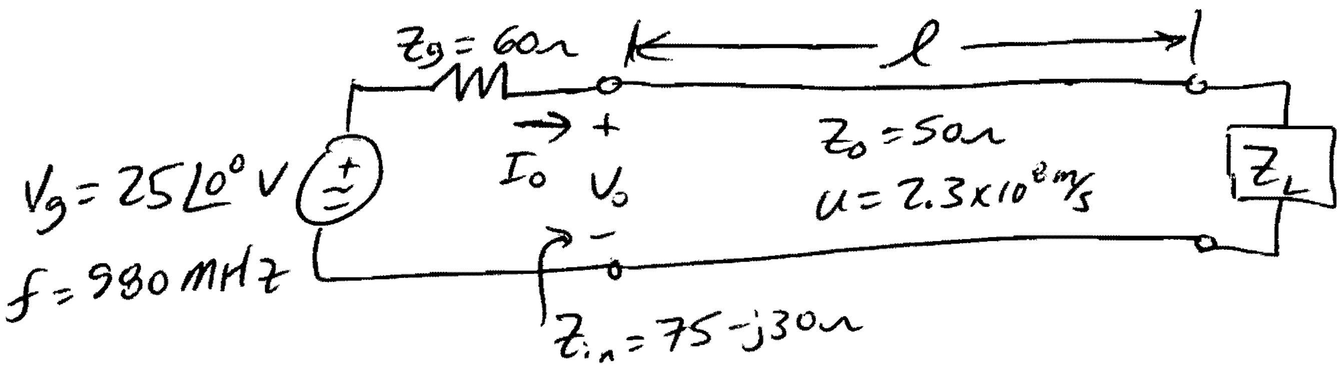
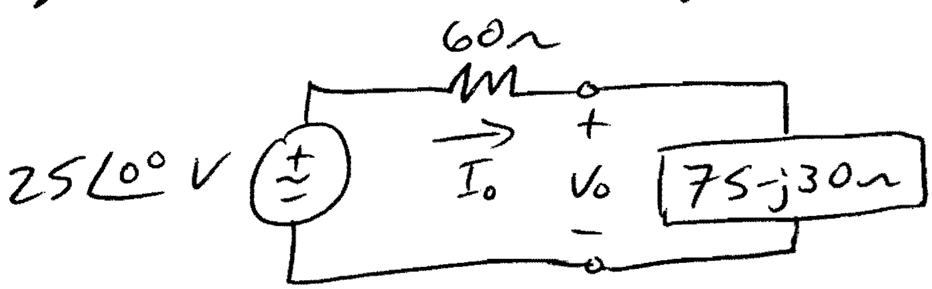
A lossless transmission line (Z_0 = 50 Ω, u = 2.3×10^8 m/s) of some length l is terminated with an unknown load Z_L. Using a vector network analyzer (VNA), an input impedance Z_{in} = 75 - j30 Ω is measured. The transmission line is connected to a generator operating at 980 MHz with a voltage $v_g(t) = 25\cos(\omega t)$ V and impedance Z_g = 60 Ω. Draw the transmission line circuit. Then, determine (a) the phase constant β & wavelength λ for the transmission line, (b) the phasor current I_0 & voltage V_0 at the input, (c) the input reflection coefficient Γ_{in}, (d) the phasor forward V_0^+ & backward V_0^- voltages, (e) the equations for the phasor current $I_s(z)$ & voltage $V_s(z)$ along the transmission line, and (f) the time-domain equations for the current $I(z,t)$ & voltage $V(z,t)$ along the transmission line.



a) $\beta = \omega / u = \dfrac{2\pi(980\times10^6)}{2.3\times10^8} = \underline{26.772 \text{ rad/m}}$

$\lambda = u/f = \dfrac{2.3\times10^8}{980\times10^6} = \underline{0.23469 \text{ m}}$

b) Equivalent Circuit @ input



$$I_0 = \frac{25\angle 0^\circ}{60 + (75 - j30)} = \underline{0.180775\angle 12.5288^\circ \text{ A}}$$

$$V_0 = (25\angle 0^\circ)\frac{75 - j30}{60 + (75 - j30)} = \underline{14.6026\angle -9.273^\circ \text{ V}}$$

c) $\Gamma_{in} = \dfrac{Z_{in} - Z_0}{Z_{in} + Z_0} = \dfrac{(75 - j30) - 50}{(75 - j30) + 50}$

$\Gamma_{in} = 0.3038 \angle -36.699°$

d) From $V_s(0) = V_0 = V_0^+ e^{-j\beta(0)} (1 + \Gamma(0))$, where $e^{-j\beta(0)} \to 1$ and $\Gamma(0) \to \Gamma_{in}$

$\to V_0^+ = \dfrac{V_0}{1 + \Gamma_{in}} = \dfrac{14.603 \angle -9.273°}{1 + 0.3038 \angle -36.7°}$

$V_0^+ = 11.619 \angle -0.967° \ V$

$\Gamma_{in} = \dfrac{V_0^-}{V_0^+} e^0 \Rightarrow V_0^- = \Gamma_{in} V_0^+ = (0.3038 \angle -36.7°)(11.619 \angle -0.967°)$

$V_0^- = 3.5298 \angle -37.6656° \ V$

e) $V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$

$V_s(z) = (11.619 \angle -0.967°) e^{-j26.772z} + (3.53 \angle -37.7°) e^{j26.772z} \ (V)$

$0 \le z \le \ell$

$I_s(z) = \dfrac{V_0^+}{Z_0} e^{-j\beta z} - \dfrac{V_0^-}{Z_0} e^{j\beta z}$

$= \left(\dfrac{11.619 \angle -0.967°}{50}\right) e^{-j26.772z} - \left(\dfrac{3.53 \angle -37.7°}{50}\right) e^{j26.772z}$

$I_s(z) = (0.2324 \angle -0.967°) e^{-j26.772z} - (0.0706 \angle -37.7°) e^{j26.772z} \ (A)$

$0 \le z \le \ell$

f) $V(z,t) = \mathrm{Re}\{V_s(z)\, e^{j\omega t}\}$

$$V(z,t) = 11.619 \cos(\omega t - 26.772z - 0.967^\circ) + 3.53 \cos(\omega t + 26.772z - 37.7^\circ)\ \text{V}$$

where $\omega = 2\pi(980\times10^6) = 6.1575\times10^9$ rad/s $\quad 0 \le z \le \ell$

$I(z,t) = \mathrm{Re}\{I_s\, e^{j\omega t}\}$

$$I(z,t) = 0.2324 \cos(\omega t - 26.772z - 0.967^\circ) - 0.0706 \cos(\omega t + 26.772z - 37.7^\circ)\ \text{A}$$

where $\omega = 6.1575\times10^9$ rad/s $\quad 0 \le z \le \ell$