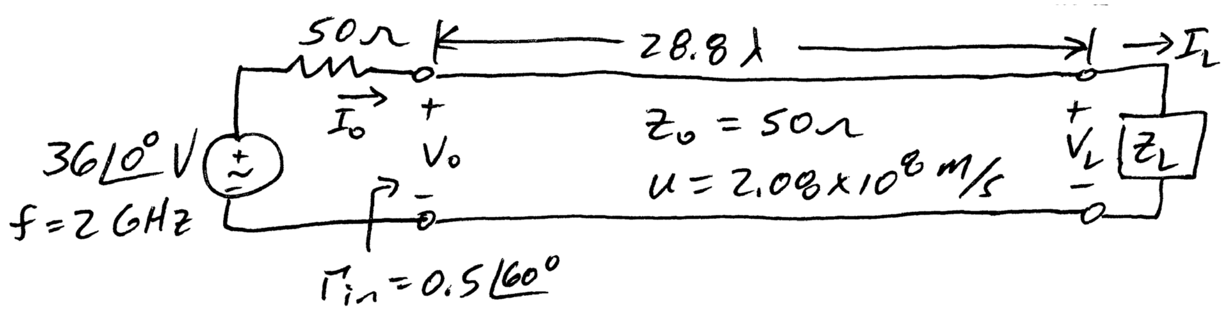
A lossless transmission line ($Z_0 = 50\ \Omega$, $u = 2.08 \times 10^8$ m/s) of length 28.8λ is terminated with an unknown load. Using a vector network analyzer (VNA), an input reflection coefficient of $\Gamma_{in} = 0.50\angle 60°$ is measured. The transmission line (TL) is then connected to a generator with a voltage $36\angle 0°$ V and impedance 50 Ω operating at 2 GHz. Draw the TL circuit. Then, determine the (a) propagation constant & wavelength, (b) input impedance, (c) phasor current & voltage and time-average power at the input, (d) phasor forward voltage wave amplitude, (e) phasor current & voltage and time-average power at the load.



a) lossless $\Rightarrow \alpha = 0$

(11.21) $\beta = \omega/u = \dfrac{2\pi(2\times10^9)}{2.08\times10^8} \Rightarrow \beta = 60.4152$ rad/m

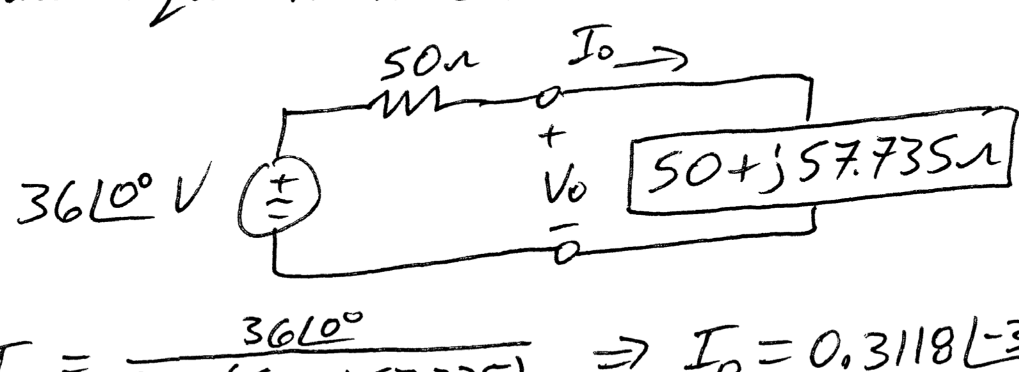
(11.11) $\gamma = \alpha + j\beta \Rightarrow \underline{\gamma = j60.4152\ \text{m}^{-1}}$

(11.14) $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{60.415} \Rightarrow \underline{\lambda = 0.104\ \text{m}}$

b) From notes $Z_{in} = Z_0 \dfrac{1+\Gamma_{in}}{1-\Gamma_{in}}$

$Z_{in} = 50\dfrac{1+0.5\angle 60°}{1-0.5\angle 60°} \Rightarrow \underline{Z_{in} = 50 + j57.735\ \Omega}$

c) Draw equivalent circuit



$I_0 = \dfrac{36\angle 0°}{50+(50+j57.735)} \Rightarrow \underline{I_0 = 0.3118\angle{-30°}\ A}$

c) cont

$$V_0 = 36\angle 0^\circ \frac{50+j57.735}{50+50+j57.735} \Rightarrow \underline{\underline{V_0 = 23.8118\angle 19.107^\circ \, V}}$$

$$P_{in} = \tfrac{1}{2}\,\mathrm{Re}\{V_0 I_0^*\} = \tfrac{1}{2}\,\mathrm{Re}\{23.81\angle 19.11^\circ (0.312\angle +30^\circ)\}$$

$$\underline{\underline{P_{in} = 2.43 \text{ W}}}$$

d) (11.27) $V_0^+ = \tfrac{1}{2}\left[V_0 + I_0 Z_0\right]$

$$= \tfrac{1}{2}\left[23.81\angle 19.11^\circ + (0.312\angle -30^\circ)\,50\right]$$

$$\underline{\underline{V_0^+ = 18\angle 0^\circ \, V}}$$

e) Notes $\Gamma_L = \Gamma_{in}\, e^{j2\beta\ell} = (0.5\angle 60^\circ)\, e^{j2\frac{2\pi}{\lambda}28.8\lambda}$

$$\Gamma_L = 0.5\angle -84^\circ$$

$$I_L = \frac{V_0^+}{Z_0} e^{-j\beta\ell}(1-\Gamma_L) = \frac{18\angle 0^\circ}{50} e^{-j\frac{2\pi}{\lambda}28.8\lambda}(1-0.5\angle -84^\circ)$$

$$\underline{\underline{I_L = 0.3853\angle 99.685^\circ \, A}}$$

$$V_L = V_0^+ e^{-j\beta\ell}(1+\Gamma_L) = 18\angle 0^\circ\, e^{-j\frac{2\pi}{\lambda}28.8\lambda}(1+0.5\angle -84^\circ)$$

$$\underline{\underline{V_L = 20.9492\angle 46.706^\circ \, V}}$$

$$P_L = \tfrac{1}{2}\,\mathrm{Re}\{V_L I_L^*\} = \tfrac{1}{2}\,\mathrm{Re}\{20.95\angle 46.7^\circ (0.385\angle -99.7^\circ)\}$$

$$\underline{\underline{P_L = P_{in} = 2.43 \text{ W}}}$$