For the novel coaxial transmission line, find a) the propagation constant γ, attenuation constant α (Np/m & dB/m), phase constant β, wavelength λ, phase velocity u, and characteristic impedance Z_0. b) Is this coaxial transmission line low loss? Why or why not? c) Regardless, calculate α, β, u, and Z_0 using the low loss approximations. In a table, compare the exact values of α, β, u, and Z_0 to the low loss approximations. Table format: Col. 1 variable, col. 2 exact value, and col. 3 low loss approximate value.

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A novel coaxial transmission line is made with lead conductors, a center wire of diameter 2 mm and shield of diameter 10 mm, separated by a Bakelite insulator (AKA: polyoxybenzylmethylenglycolanhydride). It is connected to a signal generator operating at 103.5 MHz. Find the skin depth as well as the resistance per-unit-length R, inductance per-unit-length L, conductance per-unit-length G, and capacitance-per-unit-length C.

Prior: $R = 1.7265\ \Omega/\text{m}$, $L = 321.888$ nH/m, $G = 390.396$ pS/m, & $C = 172.832$ pF/m.

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a) Used MathCAD for precision in all calculations.

Per (11.11),

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

$$= \sqrt{[1.7265 + j(2\pi 103.5 \cdot 10^{6})321.888 \cdot 10^{-9}][390.396 \cdot 10^{-12} + j(2\pi 103.5 \cdot 10^{6})172.832 \cdot 10^{-12}]}$$

$$\Rightarrow \quad \underline{\boldsymbol{\gamma = 0.020003 + j\,4.850519\ \text{1/m}}}$$

$$\alpha = \text{Re}(\gamma) \quad \Rightarrow \quad \underline{\boldsymbol{\alpha = 0.020003\ \text{Np/m}}}$$

$$\alpha = 0.020003\ \text{Np/m}\ (8.6859\ \text{dB/Np}) \quad \Rightarrow \quad \underline{\boldsymbol{\alpha = 0.17374\ \text{dB/m}}}$$

$$\beta = \text{Im}(\gamma) \quad \Rightarrow \quad \underline{\boldsymbol{\beta = 4.85052\ \text{rad/m}}}$$

Per (11.13), $\lambda = 2\pi/\beta = 2\pi/4.85052 \quad \Rightarrow \quad \underline{\boldsymbol{\lambda = 1.29536\ \text{m}}}$

Per (11.14), $u = \omega/\beta = 2\pi(103.5\times 10^{6})/\ 4.85052 \quad \Rightarrow \quad \underline{\boldsymbol{u = 1.340701 \times 10^{8}\ \text{m/s}}}$

Per (11.19), $Z_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}} = \sqrt{\dfrac{1.7265 + j(2\pi 103.5 \cdot 10^{6})1.0338 \cdot 10^{-6}}{3.904 \cdot 10^{-10} + j(2\pi 103.5 \cdot 10^{6})1.7283 \cdot 10^{-10}}}$

$$\Rightarrow \quad \underline{\boldsymbol{Z_0 = 43.1562 - j\,0.177797\ \Omega}}$$

b) Per notes, check to see if $R << \omega L$ and $G << \omega C$ as conditions for low loss transmission line (TL).

Is $R = 1.726\ \Omega/\text{m} << \omega L = 2\pi\,(103.5 \times 10^6)\, 3.219 \times 10^{-7} = 209.33\ \Omega/\text{m}$? **YES.**

Is $G = 390.4\ \text{pS/m} << \omega C = 2\pi\,(103.5 \times 10^6)\, 1.728 \times 10^{-10} = 0.1124\ \text{S/m}$? **YES.**

$\Rightarrow$ **<u>This TL is low loss as it meets both the $R << \omega L$ and $G << \omega C$ conditions.</u>**

c) Use low loss approximation equations (notes) with MathCAD for precision-

$$\alpha \simeq \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right] = 0.5\left[1.73\sqrt{\frac{172.8\times10^{-12}}{321.9\times10^{-9}}} + 390.4\times10^{-12}\sqrt{\frac{321.9\times10^{-9}}{172.8\times10^{-12}}}\right]$$

$\Rightarrow$ **$\alpha \approx$ 0.0200029 Np/m**

$$\beta \simeq \omega\sqrt{LC} = 2\pi(103.5\times10^{6})\sqrt{321.9\times10^{-9}(172.8\times10^{-12})}$$

$\Rightarrow$ **$\beta \approx$ 4.8504778 rad/m**

$$u \simeq \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{321.9\times10^{-9}(172.8\times10^{-12})}}$$

$\Rightarrow$ **$u \approx 1.340713 \times 10^8$ m/s,**

and

$$Z_0 = R_0 + jX_0 \simeq \sqrt{\frac{L}{C}} - j\sqrt{\frac{L}{C}}\frac{1}{2\omega}\left(\frac{R}{L} - \frac{G}{C}\right)$$

$$= \sqrt{\frac{321.9\times10^{-9}}{172.8\times10^{-12}}} - j\sqrt{\frac{321.9\times10^{-9}}{172.8\times10^{-12}}}\frac{1}{2(2\pi)103.5\times10^{6}}\left(\frac{1.7265}{321.9\times10^{-9}} - \frac{390.4\times10^{-12}}{172.8\times10^{-12}}\right)$$

$\Rightarrow$ **$Z_0 \approx 43.155875 - j\, 0.17797\ \Omega$.**

Variable	Exact	Low loss approximation
α	0.02000273 Np/m	0.0200029 Np/m
α	0.17374152 dB/m	0.17374152 Np/m
β	4.850519 rad/m	4.8504778 rad/m
u	1.340701×10^8 m/s	1.340713×10^8 m/s
Z_0	43.156242 - j 0.177969 Ω	43.155875 - j 0.177971 Ω

<u>The answers agree to 5^+ significant digits in the case of this low loss TL!</u>