For the twin-lead transmission line, find a) the propagation constant γ, attenuation constant α (Np/m & dB/m), phase constant β, wavelength λ, phase velocity u, and characteristic impedance Z_0. b) Is the twin-lead transmission line low loss? Why or why not? Regardless, in a table, compare the exact values of α, β, u, and Z_0 from those already found to those calculated using the low loss approximations. Table format: Col. 1 variable, col. 2 exact value, col. 3 low loss approximate value, and col. 4 percent difference.

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A twin-lead transmission line is made of brass wires with diameters of 1.2 mm, separated by 8 mm center-to-center, embedded in a dielectric insulator characterized by $\varepsilon = 1.6\varepsilon_0$, $\mu = \mu_0$, and $\sigma = 5 \times 10^{-6}$ S/m. Determine the skin depth δ and per-unit-length parameters R, L, C, and G at a frequency of 530 MHz.

From earlier- R = 7.3168 Ω /m, L = 1.0338 μH/m, G = 6.0775 μS/m, & C = 17.209 pF/m.

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a) Used MathCAD for precision in all calculations.

Per (11.11),

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

$$= \sqrt{[7.3168 + j(2\pi 530 \cdot 10^6)1.0338 \cdot 10^{-6}][6.0775 \cdot 10^{-6} + j(2\pi 530 \cdot 10^6)17.209 \cdot 10^{-12}]}$$

$$\Rightarrow \quad \underline{\boldsymbol{\gamma = 0.015675 + j\ 14.05046\ 1/m}}$$

$$\alpha = \mathrm{Re}(\gamma) \quad \Rightarrow \quad \underline{\boldsymbol{\alpha = 0.01567502\ Np/m}}$$

$$\alpha = 0.01568 \text{Np/m } (8.6859 \text{ dB/Np}) \quad \Rightarrow \quad \underline{\boldsymbol{\alpha = 0.1361515\ dB/m}}$$

$$\beta = \mathrm{Im}(\gamma) \quad \Rightarrow \quad \underline{\boldsymbol{\beta = 14.050463\ rad/m}}$$

Per (11.13), $\lambda = 2\pi/\beta = 2\pi/14.05046 \quad \Rightarrow \quad \underline{\boldsymbol{\lambda = 0.447187\ m}}$

Per (11.14), $u = \omega/\beta = 2\pi(530 \times 10^6)/\ 14.05046 \quad \Rightarrow \quad \underline{\boldsymbol{u = 2.370091 \times 10^8\ m/s}}$

Per (11.19), $Z_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}} = \sqrt{\dfrac{7.3168 + j(2\pi 530 \cdot 10^6)1.0338 \cdot 10^{-6}}{6.0775 \cdot 10^{-6} + j(2\pi 530 \cdot 10^6)17.209 \cdot 10^{-12}}}$

$$\Rightarrow \quad \underline{\boldsymbol{Z_0 = 245.029247 - j\ 0.24739\ \Omega}}$$

b) Per notes, check to see if $R << \omega L$ and $G << \omega C$ as conditions for low loss transmission line (TL).

Is $R = 7.3\ \Omega/\text{m} << \omega L = 2\pi\,(530\times10^{6})\,1.0338\times10^{-6} = 3442.8\ \Omega/\text{m}$? **YES.**

Is $G = G = 6.08\ \mu\text{S/m} << \omega C = 2\pi\,(530\times10^{6})\,17.2\times10^{-12} = 0.0573\ \text{S/m}$? **YES.**

$\Rightarrow$ **<u>This TL is low loss as it meets both the $R << \omega L$ and $G << \omega C$ conditions</u>.**

Per low loss approximations (notes) and using MathCAD for precision-

$$\alpha \simeq \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right] = 0.5\left[7.3\sqrt{\frac{17.2\times10^{-12}}{1.0\times10^{-6}}} + 6.08\times10^{-6}\sqrt{\frac{1.0\times10^{-6}}{17.2\times10^{-12}}}\right]$$

$\Rightarrow \boldsymbol{\alpha \approx 0.01567503\ \text{Np/m}}$

$$\beta \simeq \omega\sqrt{LC} = 2\pi(530\times10^{6})\sqrt{1.0338\times10^{-6}(17.209\times10^{-12})} \qquad \Rightarrow \boldsymbol{\beta \approx 14.050456\ \text{rad/m}}$$

$$u \simeq \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.0338\times10^{-6}(17.209\times10^{-12})}} \qquad \Rightarrow \boldsymbol{u \approx 2.370093\times10^{8}\ \text{m/s}},$$

and

$$Z_0 = R_0 + jX_0 \simeq \sqrt{\frac{L}{C}} - j\sqrt{\frac{L}{C}}\frac{1}{2\omega}\left(\frac{R}{L} - \frac{G}{C}\right)$$

$$= \sqrt{\frac{1.03\times10^{-6}}{17.2\times10^{-12}}} - j\sqrt{\frac{1.03\times10^{-6}}{17.2\times10^{-12}}}\frac{1}{2(2\pi)530\times10^{6}}\left(\frac{7.32}{1.0\times10^{-6}} - \frac{6.08\times10^{-6}}{17.2\times10^{-12}}\right)$$

$\Rightarrow\ \boldsymbol{Z_0 \approx 245.029096 - j\ 0.24739\ \Omega}$.

Variable	Exact	Low loss approximation	% diff.
α	0.01567502 Np/m	0.01567503 Np/m	0.000051
α	0.1361515 dB/m	0.1361515 Np/m	0
β	14.050463 rad/m	14.050456 rad/m	0.000051
u	2.370091×10^{8} m/s	2.370093×10^{8} m/s	0.000051
Z_0	245.029247 - j 0.24739 Ω	245.029096 - j 0.24739 Ω	0.000062*

*** Compared |Z0| values**

<u>The answers agree to within 0.00005% - 0.00006% in the case of this low loss TL</u>!