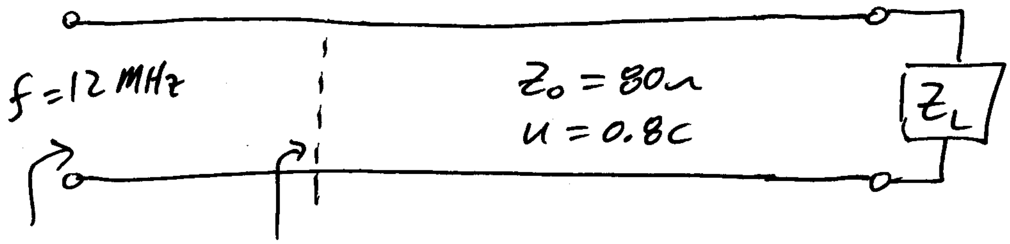
11.49 An 80 Ω transmission line operating at 12 MHz is terminated by a load Z_L. At 22 m from the load, the input impedance is $100 - j120$ Ω. If $u = 0.8c$,

(a) Calculate Γ_L, $Z_{\text{in, max}}$, and $Z_{\text{in, min}}$.
(b) Find Z_L, s, and the input impedance at 28 m from the load.
(c) How many $Z_{\text{in, max}}$ and $Z_{\text{in, min}}$ are there between the load and the $100 - j120$ Ω input impedance?

- Solve this problem analytically, don't use Smith chart.



$Z_{in}(\ell' = 28\text{m})$ $\quad Z_{in}(\ell' = 22\text{m}) = 100 - j120\,\Omega$

$$\beta = \frac{\omega}{u} = \frac{2\pi(12\times10^6)}{0.8(2.9979\times10^8)} = 0.31438\ \frac{\text{rad}}{\text{m}} \qquad (11.21a)$$

$$(11.37)\quad \Gamma_{in,22} = \frac{Z_{in,22} - Z_0}{Z_{in,22} + Z_0} = \frac{(100 - j120) - 80}{(100 - j120) + 80} = 0.56235\angle{-46.848^\circ}$$

$$\text{VSWR} = S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 3.570 \qquad (11.38a)$$

a) $$\Gamma_L = \Gamma_{in,22}\, e^{j2\beta\ell'} = 0.56235\angle{-46.848^\circ}\, e^{j2(0.3144)22}$$

$$\hookrightarrow \underline{\underline{\Gamma_L = 0.56235\angle{25.701^\circ}}}$$

$$(11.39a)\quad Z_{in,max} = S Z_0 = 3.57(80) \Rightarrow \underline{\underline{Z_{in,max} = 285.59\,\Omega}}$$

$$(11.39b)\quad Z_{in,min} = Z_0/S = \frac{80}{3.57} \Rightarrow \underline{\underline{Z_{in,min} = 22.41\,\Omega}}$$

b) (11.36) & notes

$$Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L} = 80\, \frac{1 + 0.56\angle{25.7^\circ}}{1 - 0.56\angle{25.7^\circ}} \Rightarrow \underline{\underline{Z_L = 180.65 + j128.86\,\Omega}}$$

b) cont. Per earlier calculation, $\underline{\underline{S = 3.570}}$

(11.34) $$Z_{in}(\ell' = 28m) = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta \ell')}{Z_0 + jZ_L \tan(\beta \ell')} \right]$$

$$= 80 \left[\frac{(180.65 + j128.86) + j80 \tan(0.3144(28))}{80 + j(180.65 + j128.86) \tan(0.3144(28))} \right]$$

$$\underline{\underline{Z_{in}(\ell' = 28m) = 37.637 + j61.447\ \Omega}}$$

c) First find λ & $\lambda/2$

$$\lambda = \frac{u}{f} = \frac{0.8(2.9979 \times 10^8)}{12 \times 10^6} = 19.986\ m$$

$$\frac{\ell}{\lambda} = \frac{22m}{19.986} = 1.101 \quad \text{or} \quad \frac{\ell}{\lambda/2} = 2.201$$

$\Rightarrow$ we will see at least <u>two</u> min + max

$Z_{in,max}$ occur when $\Gamma(z) = \Gamma_{max} = |\Gamma| e^{j\theta_{max}} = \Gamma_L e^{-j2\beta\ell'}$

where $\theta_{max} = 0, -2\pi, -4\pi, \ldots = \theta_{\Gamma_L} - 2\beta\ell' = \theta_{\Gamma_L} - 2\beta(22 - z_{max})$

$$\theta_{max} = 25.701^\circ \left(\frac{\pi}{180^\circ}\right) - 2(0.3144)(22 - z_{max})$$

#1 $\theta_{max,1} = 0 \Rightarrow z_{max,1} = \dfrac{0 - 25.7(\pi/180) + 2(0.3144)22}{2(0.3144)} = 21.287\ m$

#2 $\theta_{max,2} = -2\pi \Rightarrow z_{max,2} = \dfrac{-2\pi - 25.7(\pi/180) + 2(0.3144)22}{2(0.3144)} = 11.294\ m$

#3 $\theta_{max,3} = -4\pi \Rightarrow z_{max,3} = \dfrac{-4\pi - 25.7(\pi/180) + 2(0.3144)22}{2(0.3144)} = 1.3\ m$

$\Rightarrow$ <u>There are three (3) $Z_{in,max}$</u>

c) $Z_{in,min}$ occur when $\Gamma(z) = \Gamma_{min} = |\Gamma| e^{j\theta_{min}} = \Gamma_L e^{-j2\beta\ell'}$

where $\theta_{min} = -\pi, -3\pi, \ldots = \theta_{\Gamma_L} - 2\beta\ell' = \theta_{\Gamma_L} - 2\beta(22 - z_{min})$

$$\theta_{min} = 25.701^\circ\left(\pi/180^\circ\right) - 2(0.3144)(22 - z_{min})$$

#1 $\theta_{min_1} = -\pi \Rightarrow z_{min_1} = \dfrac{-\pi - 25.7(\pi/180) + 2(0.3144)22}{2(0.3144)} = 16.29\,m$

#2 $\theta_{min_2} = -3\pi \Rightarrow z_{min_2} = \dfrac{-3\pi - 25.7(\pi/180) + 2(0.3144)22}{2(0.3144)} = 6.3\,m$

#3 $\theta_{min_3} = -5\pi \Rightarrow z_{min_3} = \dfrac{-5\pi - 25.7(\pi/180) + 2(0.3144)22}{2(0.3144)} = -3.7\,m$

NO!

$\Rightarrow$ There are two (2) $Z_{in,min}$