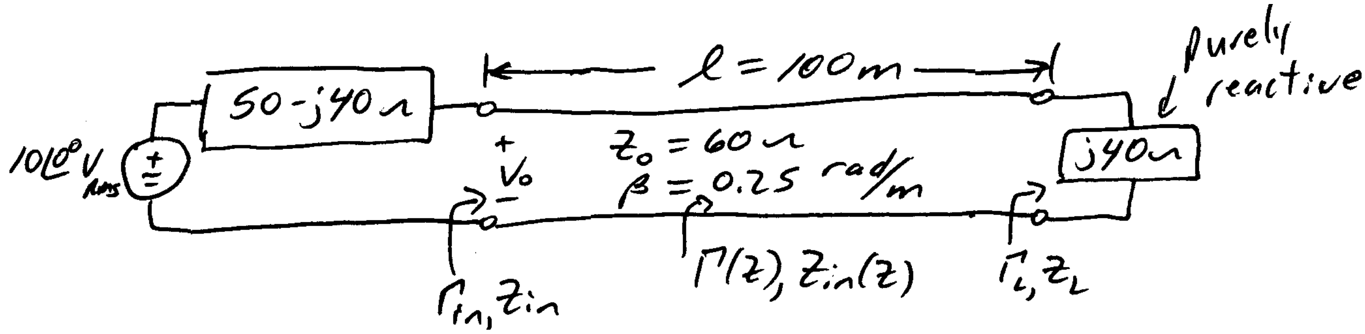
11.33 A 60 Ω lossless line is connected to a source with $V_g = 10\angle 0° \ V_{rms}$ and $Z_g = 50 - j40 \ \Omega$ and terminated with a load of $j40 \ \Omega$. If the line is 100 m long and $\beta = 0.25$ rad/m, calculate Z_{in} and V at

(a) The sending end
(b) The receiving end
(c) 4 m from the load
(d) 3 m from the source

- Note that generator voltage is in RMS. Before parts b)-d), find V_0^+, V_0^-, & $V_S(z)$ in RMS. **Hint:** What the text calls V is $V_S(z)$.



$$(11.36) \quad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{j40 - 60}{j40 + 60} = 1\angle 112.6199°$$

$$\Gamma(z) = \Gamma_L e^{-j2\beta(\ell - z)} = (1\angle 112.62°)\, e^{-j0.5(100 - z)}$$

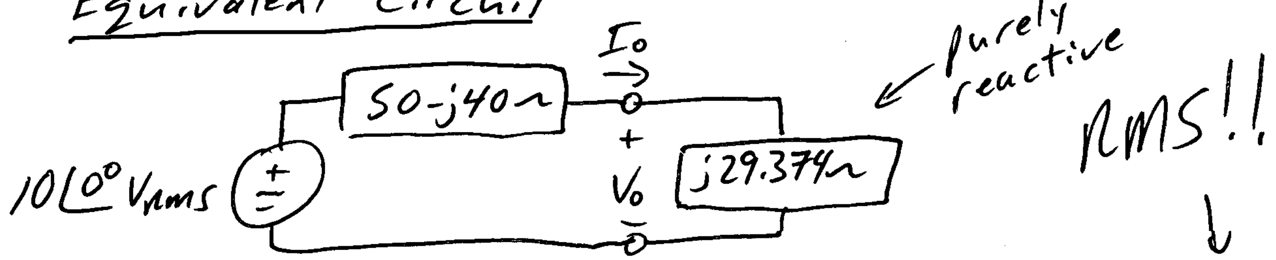
a) sending end ⇒ $z = 0$

$$\Gamma_{in} = \Gamma(0) = (1\angle 112.62°)\, e^{-j0.5(100)} = 1\angle 127.8309°$$

$$Z_{in}(0) = Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 60 \, \frac{1 + 1\angle 127.83°}{1 - 1\angle 127.83°}$$

$$\underline{\underline{Z_{in}(0) = Z_{in} = j29.374 \ \Omega}}$$

<u>Equivalent Circuit</u>



$$V_0 = V_S(0) = 10\angle 0° \, \frac{j29.374}{50 - j40 + j29.374} = \underline{\underline{5.746\angle 101.998° \ V_{rms}}}$$

$$I_0 = \frac{10\angle 0°}{50 - j40 + j29.374} = 0.19563\angle 11.998° \ A_{rms}$$

Before parts b)-d), find V_0^+, V_0^-, and $V_s(z)$.

(11.27a) $V_0^+ = \frac{1}{2}[V_0 + I_0 Z_0]$ or $\frac{V_0}{1+\Gamma_{in}} = \frac{5.746\angle 102^\circ}{1+1\angle 127.83^\circ}$

$$V_0^+ = 6.5345\angle 38.083^\circ \text{ V}_{rms}$$

(11.27b) $V_0^- = \frac{1}{2}[V_0 - I_0 Z_0]$ or $V_0 - V_0^+ = (5.746\angle 102^\circ) - 6.53\angle 38^\circ$

$$V_0^- = 6.5345\angle 165.914^\circ \text{ V}_{rms}$$

(11.24) $V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$

$$V_s(z) = (6.5345\angle 38.08^\circ)e^{-j0.25z} + (6.5345\angle 165.91^\circ)e^{j0.25z} \text{ V}_{rms}$$

$$0 \le z \le 100\text{m}$$

b) receiving end ($z = \ell = 100$m)

$$Z_{in}(100\text{m}) = Z_L = j40\,\Omega \quad \text{(given)}$$

$$V_s(100\text{m}) = V_L = (6.53\angle 38^\circ)e^{-j0.25(100)} + (6.53\angle 166^\circ)e^{j0.25(100)}$$

$$V_s(100\text{m}) = V_L = 7.2494\angle 101.998^\circ \text{ V}_{rms}$$

c) 4 m from load ⇒ $z = 100 - 4 = 96$ m

$$\Gamma(96\text{m}) = (1\angle 112.62^\circ)e^{-j0.5(100-96)} = 1\angle -1.972^\circ$$

$$Z_{in}(96\text{m}) = 60\left(\frac{1 + 1\angle -1.972^\circ}{1 - 1\angle -1.972^\circ}\right)$$

$$Z_{in}(96\text{m}) = -j3486.755\,\Omega$$

c) cont.

$$V_s(96m) = (6.53\angle 38°)\,e^{-j0.25(96)} + (6.53\angle 166°)\,e^{j0.25(96)}$$

$$\underline{\underline{V_s(96m) = 13.067\angle 101.998° \ V_{rms}}}$$

d) 3 m from source ⇒ $z = 3m$

$$\Gamma(3m) = (1\angle 112.62°)\,e^{-j0.5(100-3)}$$

$$\Gamma(3m) = 1\angle -146.225°$$

$$Z_{in}(3m) = 60\left(\frac{1 + 1\angle -146.225°}{1 - 1\angle -146.225°}\right)$$

$$\underline{\underline{Z_{in}(3m) = -j18.215\,\Omega}}$$

$$V_s(3m) = (6.53\angle 38°)\,e^{-j0.25(3)} + (6.53\angle 166°)\,e^{j0.25(3)}$$

$$\underline{\underline{V_s(3m) = 3.796\angle -78.002° \ V_{rms}}}$$