11.13 A telephone line operating at 1 kHz has $R = 6.8\ \Omega$/mi, $L = 3.4$ mH/mi, $C = 8.4$ nF/mi, and $G = 0.42\ \mu$S/mi. Find (a) Z and γ, (b) phase velocity, (c) wavelength.

- First, find the parameters R, L, C, and G per **meter**.

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Per internet, 1 mi = 1609.344 m. Doing the unit conversion gives-

$R = 6.8\ \Omega/\text{mi}\ (1\ \text{mi}/1609.344\ \text{m}) \Rightarrow$ **$R = 4.22532*10^{-3}\ \Omega/\text{m} = 4.22532\ \text{m}\Omega/\text{m}$**

$L = 3.4*10^{-3}\ \text{H/mi}\ (1\ \text{mi}/1609.344\ \text{m}) \Rightarrow$ **$L = 2.11266*10^{-6}\ \text{H/m} = 2.11266\ \mu\text{H/m}$**

$G = 0.42*10^{-6}\ \text{S/mi}\ (1\ \text{mi}/1609.344\ \text{m}) \Rightarrow$ **$G = 2.60976*10^{-10}\ \text{S/m} = 260.976\ \text{pS/m}$**

$C = 8.4*10^{-9}\ \text{F/mi}\ (1\ \text{mi}/1609.344\ \text{m}) \Rightarrow$ **$C = 5.21952*10^{-12}\ \text{F/m} = 5.21952\ \text{pF/m}$**

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a) Per (11.11),

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

$$= \sqrt{[4.22532 \bullet 10^{-3} + j(2\pi \bullet 10^{3})2.11266 \bullet 10^{-6}][2.60976 \bullet 10^{-10} + j(2\pi \bullet 10^{3})5.21952 \bullet 10^{-12}]}$$

$\Rightarrow$ **$\gamma = 3.36447*10^{-6} + j\ 2.1108*10^{-5}\ 1/\text{m}$**

Per (11.19), $Z_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}} = \sqrt{\dfrac{4.22532 \bullet 10^{-3} + j(2\pi \bullet 10^{3})2.11266 \bullet 10^{-6}}{2.60976 \bullet 10^{-10} + j(2\pi \bullet 10^{3})5.21952 \bullet 10^{-12}}}$

$\Rightarrow$ **$Z_0 = 644.407 - j\ 97.4623\ \Omega$**

b) Per (11.14), $u = \omega/\beta = 2\pi(10^3)/2.1108*10^{-5} \Rightarrow$ **$u = 2.97668*10^{8}\ \text{m/s}$**

c) Per (11.13), $\lambda = 2\pi/\beta = 2\pi/2.1108*10^{-5} \Rightarrow$ **$\lambda = 2.97668*10^{5}\ \text{m}$**