8.18 A 60-turn coil carries a current of 2 A and lies in the plane $x + 2y - 5z = 12$ such that the magnetic moment **m** of the coil is directed away from the origin. Calculate **m**, assuming that the area of the coil is 8 cm^2.

- Also, determine the vector torque experienced by the coil if an external magnetic field of $\bar{H} = 30\hat{a}_z$ kA/m is applied.

Per (8.18), $\bar{m} = I S \hat{a}_n$ for a single loop

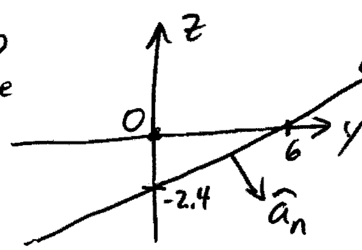
$\Rightarrow \bar{m} = N I S \hat{a}_n$ for a coil

To find $\hat{a}_n$, use the gradient function to determine vectors normal to the plane defined by the function $f = x + 2y - 5z - 12 = 0$.

$$\pm \bar{\nabla} f = \pm\left[\hat{a}_x \frac{\partial f}{\partial x} + \hat{a}_y \frac{\partial f}{\partial y} + \hat{a}_z \frac{\partial f}{\partial z}\right] = \pm\left[\hat{a}_x + 2\hat{a}_y - 5\hat{a}_z\right]$$

$x = 0$ plane

$f = \cancel{x}^{\,0} + 2y - 5z - 12 = 0$ or $z = \frac{2}{5}y - \frac{12}{5}$



From picture, we see that a vector directed away from the origin and normal to plane must have $+\hat{a}_y$ and $-\hat{a}_z$ components.

$$\hat{a}_n = \frac{+\left[\hat{a}_x + 2\hat{a}_y - 5\hat{a}_z\right]}{\sqrt{1^2 + 2^2 + (-5)^2}} = 0.1826\,\hat{a}_x + 0.3651\,\hat{a}_y - 0.9129\,\hat{a}_z$$

$$\bar{m} = 60(2)\,8\text{cm}^2\left(\frac{1\text{m}^2}{100^2\text{cm}^2}\right)\left[0.1826\,\hat{a}_x + 0.3651\,\hat{a}_y - 0.9129\,\hat{a}_z\right]$$

$$\underline{\underline{\bar{m} = 17.527\,\hat{a}_x + 35.054\,\hat{a}_y - 87.635\,\hat{a}_z \text{ mA}\cdot\text{m}^2}}$$

To determine torque, use (8.19) $\bar{T} = \bar{m} \times \bar{B}$ w/ $\bar{B} = \mu_0 \bar{H}$

$$\bar{T} = (17.527\,\hat{a}_x + 35.054\,\hat{a}_y - 87.635\,\hat{a}_z)10^{-3} \times 30\times10^{3}(4\pi\times10^{-7})\,\hat{a}_z$$

$$= -\hat{a}_y\,0.00066075 + \hat{a}_x\,0.0013215 + 0 \text{ N}\cdot\text{m}$$

$$\underline{\underline{\bar{T} = 1.3215\,\hat{a}_x - 0.66075\,\hat{a}_y \text{ mN}\cdot\text{m}}}$$