8.14 A three-phase transmission line consists of three conductors that are supported at points A, B, and C to form an equilateral triangle as shown in Figure 8.37. At one instant, conductors A and B both carry a current of 75 A while conductor C carries a return current of 150 A. Find the force per meter on conductor C at that instant.

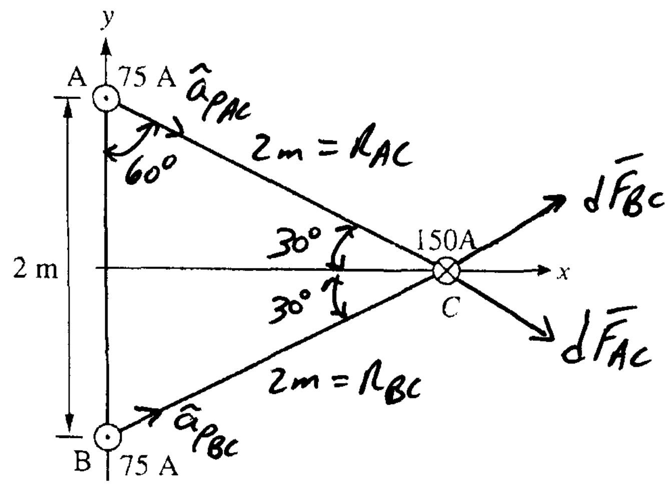

FIGURE 8.37

→ Assume 3ϕ TL is very long. Then, we can use (7.20) $\bar{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$ & (7.30) $\bar{B} = \mu_0 \bar{H}$ to get magnetic flux density vectors for conductors A & B.

→ Per (8.8), $d\bar{F} = I d\bar{\ell} \times \bar{B}$. Since we want the force per meter on conductor C, let $Id\bar{\ell}$ be

$$I_c d\bar{\ell}_c = (150A)(1m)(-\hat{a}_z) = -150\,\hat{a}_z \text{ A}\cdot\text{m}$$

Force exerted by conductor A on 1m of conductor C

→ Assume conductor A is along z-axis. Then,

$$\bar{B}_A = \frac{\mu_0 I_A}{2\pi\rho}\hat{a}_{\phi A}\Big|_{\substack{\rho = 2m \\ = R_{AC}}} = \frac{\mu_0 75}{2\pi(2)}\hat{a}_{\phi A}$$

$$d\bar{F}_{AC} = -150\,\hat{a}_{z_c} \times \frac{\mu_0 75}{4\pi}\hat{a}_{\phi A} = +\hat{a}_{\rho_{AC}}\frac{\mu_0 (150)75}{4\pi} \ (N)$$

⇒ $\hat{a}_{\rho AC}$ is vector pointing outward along the line connecting A + C as shown.

Force exerted by conductor B on 1m of conductor C

→ Assume conductor B is now along z-axis. Then,

$$\bar{B}_B = \frac{\mu_0 I_B}{2\pi\rho}\hat{a}_{\phi_B}\Big|_{\substack{\rho=2m\\=R_{BC}}} = \frac{\mu_0 75}{2\pi(2)}\hat{a}_{\phi_B}$$

$$d\bar{F}_{BC} = -150\,\hat{a}_{z_c} \times \frac{\mu_0 75}{4\pi}\hat{a}_{\phi_B} = +\hat{a}_{\rho_{BC}}\frac{\mu_0(150)75}{4\pi}\ (N)$$

where $\hat{a}_{\rho_{BC}}$ points outward along the line connecting B + C as shown.

→ Using geometry & trigonometry, express unit vectors in Cartesian coordinates

$$\hat{a}_{\rho_{AC}} = \cos 30^\circ\,\hat{a}_x - \sin 60^\circ\,\hat{a}_y$$

$$\hat{a}_{\rho_{BC}} = \cos 30^\circ\,\hat{a}_x + \sin 60^\circ\,\hat{a}_y$$

→ Adding up $d\bar{F}_{AC}$ & $d\bar{F}_{BC}$, we get

$$d\bar{F}_c = d\bar{F}_{AC} + d\bar{F}_{BC} \quad \text{for 1m length of conductor C}$$

$$= \left(\cos 30^\circ\,\hat{a}_x - \sin 60^\circ\,\hat{a}_y\right)\frac{\mu_0(150)(75)}{4\pi}$$

$$+\left(\cos 30^\circ\,\hat{a}_x + \sin 60^\circ\,\hat{a}_y\right)\frac{\mu_0(150)75}{4\pi}$$

$$= \hat{a}_x\, 2\cos 30^\circ\,\frac{4\pi\times 10^{-7}(150)75}{4\pi}$$

$$\underline{\underline{d\bar{F}_c = 1.948557\times 10^{-3}\,\hat{a}_x = 1.9486\,\hat{a}_x\ \frac{mN}{m}}}$$