PE5.7 In a dielectric material, $E_x = 5$ V/m and $\bar{P} = \frac{1}{10\pi}(3\hat{a}_x - \hat{a}_y + 4\hat{a}_z)$ nC/m^2.

Calculate: (a) χ_e, (b) $\bar{E}$, (c) $\bar{D}$.

(a) χ_e

(5.32) $\bar{P} = \chi_e \epsilon_0 \bar{E}$

$\frac{1}{10\pi}(3\hat{a}_x - \hat{a}_y + 4\hat{a}_z)\frac{nC}{m^2} = \chi_e\, 8.854\times10^{-12}(5\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z)$

equate $\hat{a}_x$ terms $\rightarrow$ $\frac{3\times10^{-9}}{10\pi} = \chi_e\, 8.854\times10^{-12}(5)$

$\chi_e = 2.15706$

(b) $\bar{E}$ $\frac{1}{10\pi}(3\hat{a}_x - \hat{a}_y + 4\hat{a}_z)\frac{nC}{m^2} = 2.157\epsilon_0(5\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z)$

equate terms $E_y = \frac{\frac{-10^{-9}}{10\pi}}{2.157\epsilon_0} = -1.\overline{66}$

$E_z = \frac{\frac{4\times10^{-9}}{10\pi}}{2.157\epsilon_0} = 6.\overline{66}$

$\bar{E} = 5\hat{a}_x - 1.\overline{66}\hat{a}_y + 6.\overline{66}\hat{a}_z$ V/m

(c) $\bar{D}$ $\bar{D} = \epsilon_0(1+\chi_e)\bar{E}$ (5.33)

$= (3.157)8.854\times10^{-12}(5\hat{a}_x - 1.66\hat{a}_y + 6.\overline{66}\hat{a}_z)$

$\bar{D} = 139.76\hat{a}_x - 46.59\hat{a}_y + 186.35\hat{a}_z$ $\frac{pC}{m^2}$