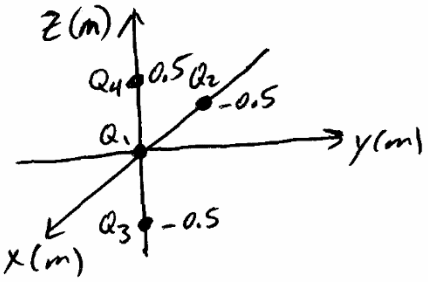
PE 4.14 mod1 Point charges $Q_1 = 1$ nC, $Q_2 = -2$ nC, $Q_3 = 3$ nC, and $Q_4 = -4$ nC are positioned one at a time and in that order at (0, 0, 0), (-0.5 m, 0, 0), (0, 0, -0.5 m), and (0, 0, 0.5 m), respectively. Calculate the energy in the system after each charge is positioned.



Use (4.87) $W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k$

where V_k is the total potential at location of Q_k due to all other charges

Charge Q_1 *Since no other charges are present $V_1 = 0$

$W_{E1} = \frac{1}{2}(1\times10^{-9})(0) = 0 \Rightarrow \underline{W_{E1} = 0}$

Charge Q_2 *Find voltage at (-0.5m, 0, 0) due to $Q_1 = 1$nC located at (0, 0, 0).

Per (4.63) $V = \frac{Q}{4\pi\epsilon_0 r}$, so $V_2 = \frac{Q_1}{4\pi\epsilon_0 r_2} = \frac{1\times10^{-9}}{4\pi(8.8542\times10^{-12})0.5}$

$V_2 = 17.97510361$ V

*Find voltage at (0, 0, 0) due to $Q_2 = -2$nC located at (-0.5m, 0, 0)

Per (4.65) $V = \frac{Q}{4\pi\epsilon_0|\bar{r}-\bar{r}'|}$, so $V_1 = \frac{Q_2}{4\pi\epsilon_0|0-\bar{r}_2|} = \frac{-2\times10^{-9}}{4\pi(8.854\times10^{-12})0.5}$

$V_1 = -35.95020722$ V

Now, $W_{E2} = \frac{1}{2}[Q_1V_1 + Q_2V_2] = \frac{1}{2}[1\times10^{-9}(-35.95) + (-2\times10^{-9})(17.975)]$

$\underline{W_{E2} = -35.9502\text{ nJ}}$

Charge Q_3

* Find voltage V_1 at (0, 0, 0) due to Q_2 and Q_3

Per (4.66), $V_1 = \frac{1}{4\pi\epsilon_0}\sum_{k=2}^{3}\frac{Q_k}{|\bar{r}-\bar{r}_k|} = \frac{1}{4\pi\,8.8542\times10^{-12}}\left[\frac{-2\times10^{-9}}{0.5} + \frac{3\times10^{-9}}{0.5}\right]$

$V_1 = 17.97510361$ V

* Find voltage V_2 at $(-0.5m, 0, 0)$ due to Q_1 & Q_3

$$V_2 = \frac{1}{4\pi\, 8.8542\times10^{-12}}\left[\frac{1\times10^{-9}}{0.5} + \frac{3\times10^{-9}}{\sqrt{0.5^2+0.5^2}}\right] = 56.106056\,58\,V$$

* Find voltage V_3 at $(0, 0, -0.5m)$ due to Q_1 & Q_2

$$V_3 = \frac{1}{4\pi\, 8.8542\times10^{-12}}\left[\frac{1\times10^{-9}}{0.5} + \frac{-2\times10^{-9}}{\sqrt{0.5^2+0.5^2}}\right] = -7.4455531703\,V$$

Now, $W_{E3} = \frac{1}{2}\left[Q_1V_1 + Q_2V_2 + Q_3V_3\right]$

$$= 0.5\left[1\times10^{-9}(17.975) - 2\times10^{-9}(56.106) + 3\times10^{-9}(-7.4455)\right]$$

$$\Rightarrow \underline{\underline{W_{E3} = -58.2868\ nJ}}$$

Charge Q_4

* Find voltage V_1 at $(0, 0, 0)$ due to Q_2, Q_3 & Q_4

$$V_1 = \frac{1}{4\pi\, 8.8542\times10^{-12}}\left[\frac{-2\times10^{-9}}{0.5} + \frac{3\times10^{-9}}{0.5} + \frac{-4\times10^{-9}}{0.5}\right] = -53.9253108\,V$$

* Find voltage V_2 at $(-0.5m, 0, 0)$ due to Q_1, Q_3 & Q_4

$$V_2 = \frac{1}{4\pi\, 8.8542\times10^{-12}}\left[\frac{1\times10^{-9}}{0.5} + \frac{3\times10^{-9}}{\sqrt{0.5^2+0.5^2}} + \frac{-4\times10^{-9}}{\sqrt{0.5^2+0.5^2}}\right] = 5.264786\,V$$

* Find voltage V_3 at $(0, 0, -0.5m)$ due to Q_1, Q_2 & Q_4

$$V_3 = \frac{1}{4\pi\, 8.8542\times10^{-12}}\left[\frac{1\times10^{-9}}{0.5} + \frac{-2\times10^{-9}}{\sqrt{0.5^2+0.5^2}} + \frac{-4\times10^{-9}}{1}\right] = -43.39574\,V$$

* Find voltage V_4 at $(0, 0, 0.5m)$ due to Q_1, Q_2, & Q_3

$$V_4 = \frac{1}{4\pi\, 8.8542\times10^{-12}}\left[\frac{1\times10^{-9}}{0.5} + \frac{-2\times10^{-9}}{\sqrt{0.5^2+0.5^2}} + \frac{3\times10^{-9}}{1}\right] = 19.5171237\,V$$

Now, $W_{E4} = \frac{1}{2}\left[Q_1V_1 + Q_2V_2 + Q_3V_3 + Q_4V_4\right]$

$$= 0.5\left[1\times10^{-9}(-53.925) - 2\times10^{-9}(5.2648) + 3\times10^{-9}(-43.396) - 4\times10^{-9}(19.517)\right]$$

$$\Rightarrow \underline{\underline{W_{E4} = -136.3553\ nJ}}$$