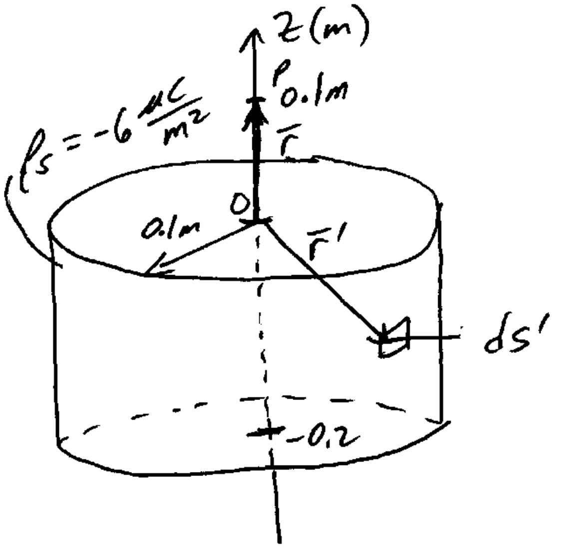
An open-ended cylinder, described by $\rho = 0.1$ m and $-0.2 \text{ m} < z < 0$, in free space supports a uniform surface charge density of -6 μC/m². Find the electric field and electric flux density vectors at point $P(0, 0, 0.1 \text{ m})$. Sketch the problem geometry with labels, including the field and source position vectors. What force will a -30 μC point charge placed at point P experience?



Surface charge density per (4.15) + class notes

$$\bar{E} = \iint_S \frac{\rho_s (\bar{r} - \bar{r}')\, ds'}{4\pi\epsilon_0 |\bar{r} - \bar{r}'|^3}$$

$$\rho_s = -6 \times 10^{-6} \text{ C/m}^2$$
$$\epsilon_0 = 8.8541878 \times 10^{-12} \text{ F/m}$$

Field position vector $\bar{r} = \underbrace{\rho}_{\to 0} \hat{a}_\rho + \underbrace{z}_{\to 0.1} \hat{a}_z = 0.1 \hat{a}_z$ (m)

Source position vector $\bar{r}' = \underbrace{\rho'}_{\to 0.1} \hat{a}_\rho' + z' \hat{a}_z = 0.1\, \hat{a}_\rho' + z' \hat{a}_z$

$$= 0.1 \left(\cos\phi' \hat{a}_x + \sin\phi' \hat{a}_y\right) + z' \hat{a}_z$$

differential surface $ds' = |d\bar{s}_\rho'| = \underbrace{\rho'}_{\to 0.1} d\phi' dz' = 0.1 d\phi' dz'$

Put them all together –

$$\bar{E}_P = \int_{z'=-0.2}^{0} \int_{\phi'=0}^{2\pi} \frac{-6\times 10^{-6} \left[0.1 \hat{a}_z - 0.1\cos\phi' \hat{a}_x - 0.1 \sin\phi' \hat{a}_y - z' \hat{a}_z\right) 0.1\, d\phi' dz'}{4\pi (8.854 \times 10^{-12}) \left| (-0.1\cos\phi')^2 + (-0.1 \sin\phi')^2 + (0.1 - z')^2 \right|^3}$$

$$= -5392.531083 \int_{z'=-0.2}^{0} \int_{\phi'=0}^{2\pi} \frac{-0.1\cos\phi' \hat{a}_x - 0.1 \sin\phi' \hat{a}_y + (0.1 - z')\hat{a}_z}{\left[0.1^2 + (0.1 - z')^2\right]^{3/2}} d\phi' dz'$$

$$\bar{E}_P = -5392.53(-0.1)\int_{z'=-0.2}^{0} \frac{dz'}{[0.1^2+(0.1-z')^2]^{3/2}} \left[\hat{a}_x \underbrace{\int_{\phi'=0}^{2\pi} \cos\phi' d\phi'}_{\to 0} + \hat{a}_y \underbrace{\int_{\phi'=0}^{2\pi} \sin\phi' d\phi'}_{\to 0}\right]$$

full cycle

$$-\hat{a}_z 5392.53108 \int_{\phi'=0}^{2\pi} d\phi' \int_{z'=-0.2}^{0} \frac{(0.1-z')\ dz'}{[0.1^2+(0.1-z')^2]^{3/2}}$$

$$= -\hat{a}_z\ 5392.53108\ (2\pi - 0) \left[\frac{1}{\sqrt{0.1^2+(0.1-z')^2}}\right]\Bigg|_{z'=-0.2}^{0}$$

$$= -\hat{a}_z 33882.272 \left[\frac{1}{\sqrt{0.1^2+0.1^2}} - \frac{1}{\sqrt{0.1^2+(0.3)^2}}\right]$$

$$\bar{E}_P = -\hat{a}_z 132{,}438.6914\ \text{V/m} = -\hat{a}_z 132.439\ \text{kV/m}$$

$$\bar{D}_P = \epsilon_0 \bar{E}_P = 8.8541878 \times 10^{-12}\left[-\hat{a}_z 132{,}438.6914\right]$$

$$\bar{D}_P = -\hat{a}_z 1.172637 \times 10^{-6}\ \text{C/m}^2 = -\hat{a}_z 1.173\ \frac{\mu C}{m^2}$$

Force on $-30\mu C$ point charge @ point P

$$\bar{F}_{-30\mu C} = q\bar{E}_P = (-30\times10^{-6})(-\hat{a}_z 132{,}438.69)$$

$$\bar{F}_{-30\mu C} = +\hat{a}_z 3.97316\ N$$