3.22 Consider the scalar function $T = r \sin\theta \cos\phi$. Determine the magnitude and direction of the maximum rate of change of T at $P(2, 6°, 30°)$.

$$\bar{\nabla} T = \hat{a}_r \frac{\partial T}{\partial r} + \hat{a}_\theta \frac{1}{r}\frac{\partial T}{\partial \theta} + \hat{a}_\phi \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}$$ spherical coordinates

$$= \hat{a}_r \frac{\partial(r\sin\theta\cos\phi)}{\partial r} + \hat{a}_\theta \frac{1}{r}\frac{\partial(r\sin\theta\cos\phi)}{\partial\theta} + \hat{a}_\phi \frac{1}{r\sin\theta}\frac{\partial(r\sin\theta\cos\phi)}{\partial\phi}$$

$$= \hat{a}_r \sin\theta\cos\phi + \hat{a}_\theta \frac{+r\cos\theta\cos\phi}{r} + \hat{a}_\phi \frac{-r\sin\theta\sin\phi}{r\sin\theta}$$

$$= \hat{a}_r \sin\theta\cos\phi + \hat{a}_\theta \cos\theta\cos\phi - \hat{a}_\phi \sin\phi$$

$$\bar{\nabla} T(2, \overset{\theta}{6°}, \overset{\phi}{30°}) = \hat{a}_r \sin 6° \cos 30° + \hat{a}_\theta \cos 6° \cos 30° - \hat{a}_\phi \sin 30°$$

$$= 0.090524305\,\hat{a}_r - 0.861281226\,\hat{a}_\theta - 0.5\,\hat{a}_\phi$$

$$|\bar{\nabla} T(2,6°,30°)| = \sqrt{\bar{\nabla}T \cdot \bar{\nabla}T} = \sqrt{(0.09052)^2 + (-0.8613)^2 + (-0.5)^2}$$

$$\underline{\underline{|\bar{\nabla} T(2,6°,30°)| = 1}}$$

$$\underline{\underline{\hat{a}_{\bar{\nabla}T} = \frac{\bar{\nabla}T}{|\bar{\nabla}T|} = 0.09052\,\hat{a}_r - 0.86128\,\hat{a}_\theta - 0.5\,\hat{a}_\phi}}$$

OR Note that $T = r\sin\theta\cos\phi = x$

In Cartesian coordinates, $\bar{\nabla}T = \bar{\nabla}(x) = \hat{a}_x$ position independent

$$\underline{\underline{|\bar{\nabla}T| = 1}} \Rightarrow \underline{\underline{\hat{a}_{\bar{\nabla}T} = \hat{a}_x}}$$