3.18 Find the gradient of the following scalar fields and evaluate the gradient at the specified point.

(a) $V(x, y, z) = 10xyz - 2x^2z$ at $P(-1, 4, 3)$

(b) $U(\rho, \phi, z) = 2\rho \sin\phi + \rho z$ at $Q(2, 90°, -1)$

(c) $W(r, \theta, \phi) = \frac{4}{r}\sin\theta\cos\phi$ at $R(1, \pi/6, \pi/2)$

a) $\bar{\nabla} V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$

$$\bar{\nabla} V = (10yz - 4xz)\hat{a}_x + 10xz\,\hat{a}_y + (10xy - 2x^2)\hat{a}_z$$

$$\bar{\nabla} V_P = \bar{\nabla} V(-1,4,3) = [10(4)3 - 4(-1)3]\hat{a}_x + 10(-1)3\hat{a}_y + [10(-1)4 - 2(-1)^2]\hat{a}_z$$

$$\bar{\nabla} V(-1,4,3) = 132\,\hat{a}_x - 30\,\hat{a}_y - 42\,\hat{a}_z$$

b) $\bar{\nabla} U = \frac{\partial U}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial U}{\partial \phi}\hat{a}_\phi + \frac{\partial U}{\partial z}\hat{a}_z$

$$\bar{\nabla} U = (2\sin\phi + z)\hat{a}_\rho + \frac{1}{\rho}(2\rho\cos\phi)\hat{a}_\phi + \rho\,\hat{a}_z$$

$$\bar{\nabla} U = (2\sin\phi + z)\hat{a}_\rho + 2\cos\phi\,\hat{a}_\phi + \rho\,\hat{a}_z$$

$$\bar{\nabla} U_Q = \bar{\nabla} U(2, 90°, -1) = [2\sin 90° + (-1)]\hat{a}_\rho + 2\overset{0}{\cancel{\cos}} 90°\,\hat{a}_\phi + 2\hat{a}_z$$

$$\bar{\nabla} U_Q = \bar{\nabla} U(2, 90°, -1) = \hat{a}_\rho + 2\hat{a}_z$$

c) $\bar{\nabla} W = \frac{\partial W}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial W}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial W}{\partial \phi}\hat{a}_\phi$

$$= \frac{-4}{r^2}\sin\theta\cos\phi\,\hat{a}_r + \frac{1}{r}\frac{4}{r}\cos\theta\cos\phi\,\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{4\sin\theta}{r}(-\sin\phi)\,\hat{a}_\phi$$

$$\bar{\nabla} W = \frac{-4}{r^2}\sin\theta\cos\phi\,\hat{a}_r + \frac{4}{r^2}\cos\theta\cos\phi\,\hat{a}_\theta - \frac{4}{r^2}\sin\phi\,\hat{a}_\phi$$

$$\bar{\nabla} W_R = \bar{\nabla} W(1, \tfrac{\pi}{6}, \tfrac{\pi}{2}) = \frac{-4}{1^2}\sin\tfrac{\pi}{6}\overset{0}{\cancel{\cos}}\tfrac{\pi}{2}\,\hat{a}_r + \frac{4}{1^2}\cos\tfrac{\pi}{6}\overset{0}{\cancel{\cos}}\tfrac{\pi}{2}\,\hat{a}_\theta - \frac{4}{1^2}\sin\tfrac{\pi}{2}\,\hat{a}_\phi$$

$$\bar{\nabla} W_R = \bar{\nabla} W(1, \tfrac{\pi}{6}, \tfrac{\pi}{2}) = -4\,\hat{a}_\phi$$