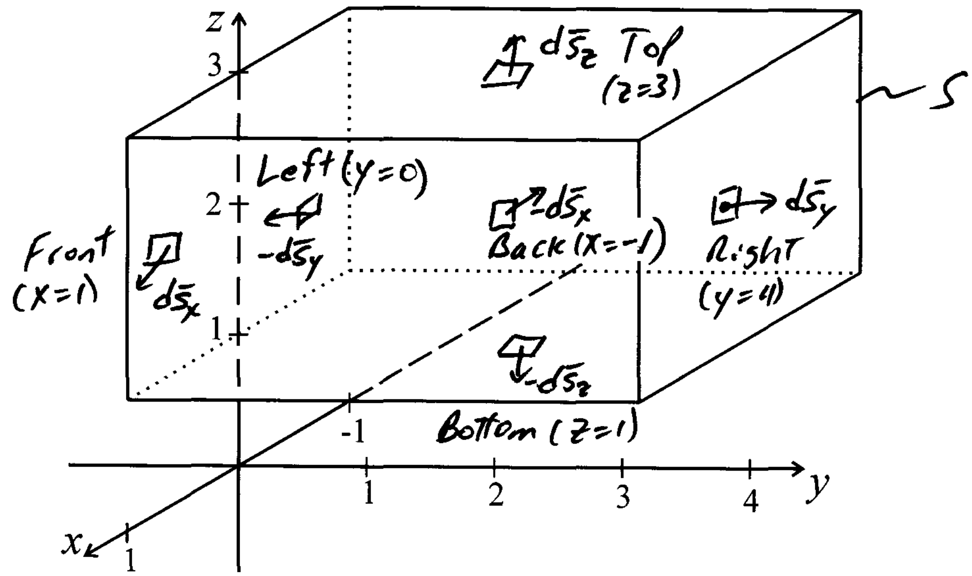
3.14 If $\mathbf{D} = x^2z\mathbf{a}_x + y^3\mathbf{a}_y + yz^2\mathbf{a}_z$, calculate the flux of $\mathbf{D}$ passing through the volume bounded by planes $x = -1$, $x = 1$, $y = 0$, $y = 4$, $z = 1$, and $z = 3$.



$\psi_D = \oint\!\!\oint_S \bar{D}\cdot d\bar{s}$ where S has $\underline{\text{six}}$ sides

$$= \iint_{\substack{\text{Front}\\(x=1)}} \bar{D}\cdot d\bar{s}_x + \iint_{\substack{\text{Back}\\(x=-1)}} \bar{D}\cdot -d\bar{s}_x + \iint_{\substack{\text{Right}\\(y=4)}} \bar{D}\cdot d\bar{s}_y + \iint_{\substack{\text{Left}\\(y=0)}} \bar{D}\cdot -d\bar{s}_y$$

$$+ \iint_{\substack{\text{Top}\\(z=3)}} \bar{D}\cdot d\bar{s}_z + \iint_{\substack{\text{Bottom}\\(z=1}} \bar{D}\cdot -d\bar{s}_z$$

$$= \int_{z=1}^{3}\int_{y=0}^{4} \cancel{\overset{1^2}{x^2}\, z}\, dy\, dz - \int_{z=1}^{3}\int_{y=0}^{4} \cancel{\overset{(-1)^2}{x^2}\, z}\, dy\, dz \quad \text{(Front + Back)} \leftarrow \text{cancel}$$

$$+ \int_{z=1}^{3}\int_{x=-1}^{1} \overset{4^3}{y^3}\, dx\, dz - \int_{z=1}^{3}\int_{x=-1}^{1} \overset{0^3}{y^3}\, dx\, dz \quad \text{(Right + Left)}$$

$$+ \int_{y=0}^{4}\int_{x=-1}^{1} y\,\overset{3^2}{z^2}\, dx\, dy - \int_{y=0}^{4}\int_{x=-1}^{1} y\,\overset{1^2}{z^2}\, dx\, dy \quad \text{(Top + Bottom)}$$

$$\Psi_0 = 64 \int_{z=1}^{3} dz \int_{x=-1}^{1} dx + 9 \int_{y=0}^{4} y\,dy \int_{x=-1}^{1} dx - \int_{y=0}^{4} y\,dy \int_{x=-1}^{1} dx$$

$$= 64(3-1)(1-(-1)) + 8\left(\frac{y^2}{2}\right)\Bigg|_{y=0}^{4} (1-(-1))$$

$$= 256 + 8\left(\frac{16}{2} - 0\right)2$$

$$\underline{\underline{\Psi_0 = 384}}$$