3.11 If $\mathbf{H} = (x - y)\mathbf{a}_x + (x^2 + zy)\mathbf{a}_y + 5yz\mathbf{a}_z$
evaluate $\int_L \mathbf{H} \cdot d\mathbf{l}$ along the contour of Figure 3.28.

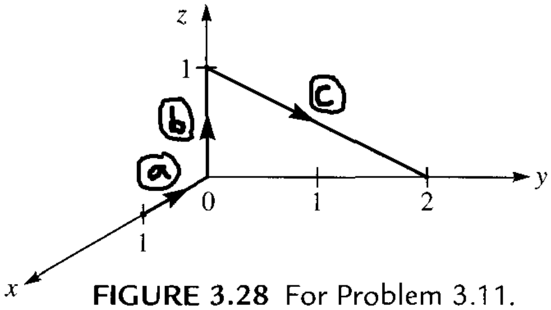

FIGURE 3.28 For Problem 3.11.

$$\bar{H}\cdot d\bar{\ell} = \left[(x-y)\hat{a}_x + (x^2+zy)\hat{a}_y + 5yz\,\hat{a}_z\right]\cdot\left(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z\right)$$

$$= (x-y)dx + (x^2+zy)dy + 5yz\,dz$$

$$\int_L \bar{H}\cdot d\bar{\ell} = \int_{a} \bar{H}\cdot d\bar{\ell} + \int_{b} \bar{H}\cdot d\bar{\ell} + \int_{c} \bar{H}\cdot d\bar{\ell}$$

a: $z=0$, $y=0$ b: $x=0$, $y=0$ c: $x=0$, $z=-\frac{1}{2}y+1$ on $y=-2(z-1) = -2z+2$

$$= \int_{x=1}^{0}(x-0)\,dx + \int_{y=0}^{0}(x^2+zy)\,dy \to 0 + \int_{z=0}^{0} 5yz\,dz \to 0 \quad \text{a}$$

$$+ \int_{x=0}^{0}(x-y)\,dx \to 0 + \int_{y=0}^{0}(x^2+zy)\,dy \to 0 + \int_{z=0}^{1} 5\cancelto{0}{y}z\,dz \to 0 \quad \text{b}$$

$$+ \int_{x=0}^{0}(x-y)\,dx \to 0 + \int_{y=0}^{2}(\cancelto{0}{x^2} + \overset{-\frac{1}{2}y+1}{z}\,y)\,dy + \int_{z=1}^{0} 5\overset{-2(z-1)}{y}\,z\,dz \quad \text{c}$$

$$= \left.\frac{x^2}{2}\right|_{x=1}^{0} + \int_{y=0}^{2}\left(-\tfrac{1}{2}y^2 + y\right)dy + \int_{z=1}^{0} 5(-2)(z^2-z)\,dz$$

$$= \left[0-\tfrac{1}{2}\right] + \left[-\frac{y^3}{6} + \frac{y^2}{2}\right]\Bigg|_{y=0}^{2} - 10\left[\frac{z^3}{3} - \frac{z^2}{2}\right]\Bigg|_{z=1}^{0}$$

$$\int_L \bar{H}\cdot d\bar{\ell} = -\tfrac{1}{2} - \tfrac{8}{6} + \tfrac{4}{2} + 10\left[\tfrac{1}{3} - \tfrac{1}{2}\right] = \underline{-1.5}$$