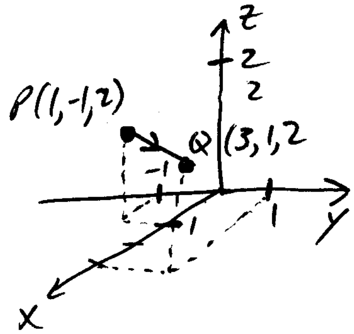
3.8 Evaluate the line integral $\int_L (2x^2 - 4xy)dx + (3xy - 2x^2y)dy$ over the straight path L joining point $P(1,-1,2)$ to $Q(3,1,2)$.



General Eq'n for line PQ

$$\frac{x-1}{3-1} = \frac{y-(-1)}{1-(-1)} = \frac{z-2}{2-2} \rightarrow \text{undefined} \quad (z = 2 \text{ constant})$$

$$\hookrightarrow x - 1 = y + 1 \Rightarrow x = y + 2$$

$$\Rightarrow y = x - 2$$

$$\int_L (2x^2-4xy)dx + (3xy-2x^2y)dy$$

$$= \int_{x=1}^{3} (2x^2 - 4x\overset{x-2}{y})dx + \int_{y=-1}^{1} (3\overset{y+2}{x}y - 2\overset{(y+2)^2}{x^2}y)dy$$

$$= \int_{x=1}^{3} (2x^2 - 4x^2 + 8x)dx + \int_{y=-1}^{1} [3y^2 + 6y - 2y(y^2+4y+4)]dy$$

$$= \int_{x=1}^{3} (-2x^2 + 8x)dx + \int_{y=-1}^{1} (-2y^3 - 5y^2 - 2y)dy$$

$$= \left(-\frac{2x^3}{3} + \frac{8x^2}{2}\right)\Bigg|_{x=1}^{3} + \left[\frac{-2y^4}{4} + \frac{-5y^3}{3} + \frac{-2y^2}{2}\right]\Bigg|_{y=-1}^{1}$$

$$= \left[\left(\frac{-2(3)^3}{3} + \frac{8(3)^2}{2}\right) - \left(\frac{-2(1)^3}{3} + \frac{8(1)^2}{2}\right)\right]$$

$$+ \left[\left(\frac{-2(1)^4}{4} + \frac{-5(1)^3}{3} + \frac{-2(1)^2}{2}\right) - \left(\frac{-2(-1)^4}{4} + \frac{-5(-1)^3}{3} + \frac{-2(-1)^2}{2}\right)\right]$$

$$= \left[(-18+36) - \left(-\frac{2}{3} + 4\right)\right] + \left\{\left(-\frac{1}{2} - \frac{5}{3} - 1\right) - \left(-\frac{1}{2} + \frac{5}{3} - 1\right)\right\}$$

$$\underline{\underline{\int_L (2x^2-4xy)dx + (3xy-2x^2y)dy = 11.\overline{3}}}$$