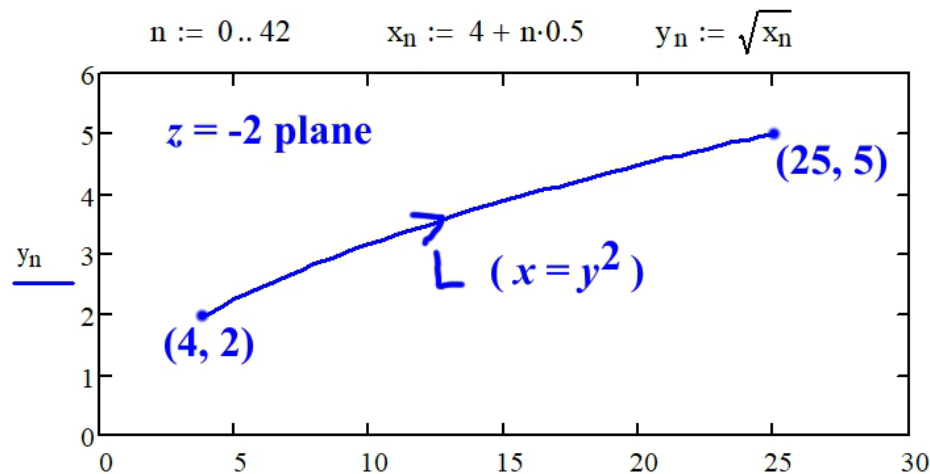


3.7 Let $\mathbf{H} = xy^2 \mathbf{a}_x + x^2y \mathbf{a}_y$. Evaluate the line integral along the parabola $x = y^2$ joining point P(4, 2, -2) to point Q(25, 5, -2).



$$\begin{aligned} \vec{H} \cdot d\vec{L} &= (xy^2 \hat{a}_x + x^2y \hat{a}_y) \cdot (dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z) \\ &= xy^2 dx + x^2y dy \end{aligned}$$

$$\int_P^Q \vec{H} \cdot d\vec{L} = \int_{x=4}^{25} x y^2 dx + \int_{y=2}^5 x^2 y dy$$

where $x = y^2$

$$= \int_{x=4}^{25} x^2 dx + \int_{y=2}^5 y^5 dy$$

$$= \left. \frac{x^3}{3} \right|_{x=4}^{25} + \left. \frac{y^6}{6} \right|_{y=2}^5$$

$$= \frac{1}{3} (25^3 - 4^3) + \frac{1}{6} (5^6 - 2^6)$$

$$= 5187 + 2593.5$$

$$\int_P^Q \vec{H} \cdot d\vec{L} = 7780.5$$

Path L ($x = y^2$)