3.3 Use the differential volume *dv* to determine the volumes of the following regions:

a) $0 < x < 1,\ 1 < y < 2,\ -3 < z < 3$ (m) ← Cartesian

$$V_a = \int_{z=-3}^{3} \int_{y=1}^{2} \int_{x=0}^{1} dx\,dy\,dz = \left(z\Big|_{-3}^{3}\right)\left(y\Big|_{1}^{2}\right)\left(x\Big|_{0}^{1}\right)$$

$$= (3+3)(2-1)(1-0)$$

$$\underline{\underline{V_a = 6\ m^3}}$$

b) $2 < \rho < 5$ m, $\pi/3 < \phi < \pi$, $-1 < z < 4$ m ← Cylindrical

$$V_b = \iiint \rho\,d\rho\,d\phi\,dz = \int_{\rho=2}^{5} \rho\,d\rho \int_{\phi=\pi/3}^{\pi} d\phi \int_{z=-1}^{4} dz$$

$$= \left(\frac{\rho^2}{2}\Big|_{2}^{5}\right)\left(\phi\Big|_{\pi/3}^{\pi}\right)\left(z\Big|_{-1}^{4}\right)$$

$$= \left(\frac{25}{2} - \frac{4}{2}\right)\left(\pi - \frac{\pi}{3}\right)(4+1)$$

$$\underline{\underline{V_b = 109.9557\ m^3}}$$

c) $1 < r < 3$ m, $\pi/2 < \theta < 2\pi/3$, $\pi/6 < \phi < \pi/2$ ← Spherical

$$V_c = \iiint r^2 \sin\theta\,dr\,d\theta\,d\phi = \int_{r=1}^{3} r^2 dr \int_{\phi=\pi/6}^{\pi/2} d\phi \int_{\pi/2}^{2\pi/3} \sin\theta\,d\theta$$

$$= \left(\frac{r^3}{3}\Big|_{1}^{3}\right)\left(\phi\Big|_{\pi/6}^{\pi/2}\right)\left(-\cos\theta\Big|_{\pi/2}^{2\pi/3}\right)$$

$$= \left(\frac{27}{3} - \frac{1}{3}\right)\left(\frac{\pi}{2} - \frac{\pi}{6}\right)\left(-\cos\frac{2\pi}{3} + \cos\frac{\pi}{2}\right)$$

$$\underline{\underline{V_c = 4.5379\ m^3}}$$