PE 2.4 If $\bar{A} = 6\hat{a}_r - 2\hat{a}_\theta + 3\hat{a}_\phi$ and $\bar{B} = 4\hat{a}_r + 3\hat{a}_\phi$, determine

(a) $\bar{A} \bullet \bar{B}$, (b) $|\bar{A} \times \bar{B}|$, (c) The vector component of $\bar{A}$ along $\hat{a}_z$ at (1, π/3, 5π/4). Assume $\bar{A}$ and $\bar{B}$ are at same point.

a) $\bar{A} \bullet \bar{B} = 6(4) + (-2)0 + 3(3) = 24 + 0 + 9$

$$\underline{\underline{\bar{A} \bullet \bar{B} = 33}}$$

b) First, find

$$\bar{A} \times \bar{B} = \left| \begin{array}{ccc|cc} \hat{a}_r & \hat{a}_\theta & \hat{a}_\phi & \hat{a}_r & \hat{a}_\theta \\ 6 & -2 & 3 & 6 & -2 \\ 4 & 0 & 3 & 4 & 0 \end{array} \right| = \begin{array}{l} \hat{a}_r[-2(3) - 3(0)] \\ + \hat{a}_\theta[3(4) - 6(3)] \\ + \hat{a}_\phi[0 - (-2)4] \end{array}$$

$$= -6\hat{a}_r - 6\hat{a}_\theta + 8\hat{a}_\phi$$

Next,

$$|\bar{A} \times \bar{B}| = \sqrt{(\bar{A} \times \bar{B}) \cdot (\bar{A} \times \bar{B})} = \sqrt{(-6)^2 + (-6)^2 + 8^2} = \sqrt{136}$$

$$\underline{\underline{|\bar{A} \times \bar{B}| = 11.6619}}$$

c) $\bar{A} \cdot \hat{a}_z = A_z = A_r \cos\theta - A_\theta \sin\theta$

$$= 6\cos\theta - (-2)\sin\theta = 6\cos\theta + 2\sin\theta$$

@ $(r = 1, \theta = \pi/3, \phi = 5\pi/4)$, $A_z = 6\cos\pi/3 + 2\sin\pi/3 = 4.73205$

In Cartesian Coordinates $\underline{\underline{\bar{A}_z = 4.732\,\hat{a}_z}}$

In spherical coordinates, use $\hat{a}_z = \cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta$

$$\bar{A}_z = 4.73205\,(\cos\overset{\pi/3}{\theta}\,\hat{a}_r - \sin\overset{\pi/3}{\theta}\,\hat{a}_\theta)$$

$$= 4.73205(0.5\,\hat{a}_r - 0.866\,\hat{a}_\theta)$$

$$\underline{\underline{\bar{A}_z = 2.366\,\hat{a}_r - 4.098\,\hat{a}_\theta}}$$