2.32 If $\mathbf{J} = r\sin\theta\cos\phi\,\mathbf{a}_r - \cos 2\theta\sin\phi\,\mathbf{a}_\theta + \tan\frac{\theta}{2}\ln r\,\mathbf{a}_\phi$ at $T(2, \pi/2, 3\pi/2)$, determine the vector component of $\mathbf{J}$ that is:

(a) Parallel to $\mathbf{a}_z$

(b) Normal to surface $\phi = 3\pi/2$

(c) Tangential to the spherical surface $r = 2$

(d) Parallel to the line $y = -2, z = 0$

At point T,

$$\bar{J}_T = 2\overset{1}{\sin\pi/2}\,\overset{0}{\cos(3\pi/2)}\,\hat{a}_r - \overset{-1}{\cos(\pi)}\,\overset{-1}{\sin(3\pi/2)}\,\hat{a}_\theta + \tan(\pi/4)\ln 2\,\hat{a}_\phi$$

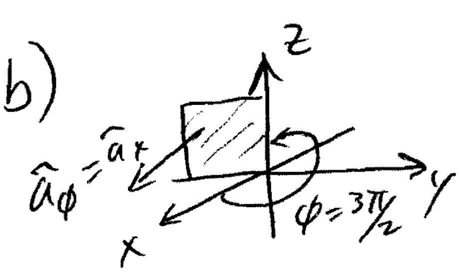
$$\bar{J}_T = -\hat{a}_\theta + 0.693147\,\hat{a}_\phi$$

a) $$\bar{J}_A = \bar{J}_{Tz} = \hat{a}_z(\bar{J}_T\cdot\hat{a}_z) = \hat{a}_z\left[(-1)(\hat{a}_\theta\cdot\hat{a}_z) + 0.693(\hat{a}_\phi\cdot\hat{a}_z)\right]$$

$$= \hat{a}_z\left[(-1)(-\sin\overset{\pi/2}{\theta}) + 0.693(0)\right]$$

$$= \hat{a}_z \overset{\text{or}}{=} \cos\overset{\pi/2}{\theta}\,\hat{a}_r - \sin\overset{\pi/2}{\theta}\,\hat{a}_\theta$$

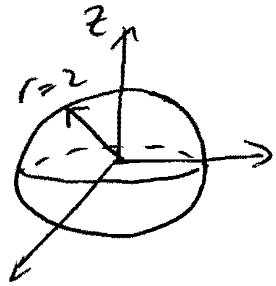
$$\underline{\underline{\bar{J}_A = \hat{a}_z = -\hat{a}_\theta}}$$ depends on which coordinate system is used

b) 

@ $\phi = 3\pi/2$ surface normal is $\hat{a}_\phi = \hat{a}_x$ (by inspection)

$$\boxed{\begin{aligned}\bar{J}_B = \bar{J}_{T\phi} &= 0.693147\,\hat{a}_\phi \quad \text{OR} \\ \bar{J}_B &= 0.693147\,\hat{a}_x\end{aligned}}$$

c)

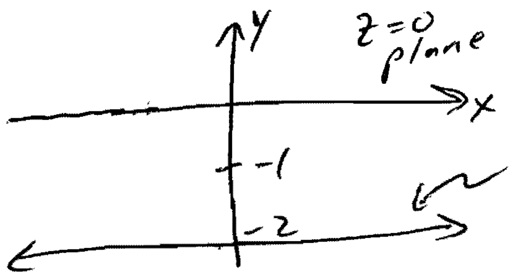


The θ & ϕ components of $\bar{J}_T$ are tangential to sphere

$$\bar{J}_c = \bar{J}_T - \cancel{\bar{J}_{T, r\text{-comp.}}}^{\,0}$$

$$\underline{\underline{\bar{J}_c = -\hat{a}_\theta + 0.69315\,\hat{a}_\phi}}$$

d) line $y=-2$ & $z=0$



the vector $\hat{a}_x$ is parallel to this line

$$\bar{J}_D = \left(\hat{a}_x \cdot \bar{J}_T\right)\hat{a}_x$$

@ $T(\underset{\uparrow\,\theta}{2, \pi/2}, \underset{\uparrow\,\phi}{3\pi/2})$, $\hat{a}_x = \sin\theta\cos\phi\,\hat{a}_r + \cos\theta\cos\phi\,\hat{a}_\theta - \sin\phi\,\hat{a}_\phi$

$$= \underbrace{\sin\tfrac{\pi}{2}}_{1}\underbrace{\cos\tfrac{3\pi}{2}}_{0}\hat{a}_r + \underbrace{\cos\tfrac{\pi}{2}}_{0}\underbrace{\cos\tfrac{3\pi}{2}}_{0}\hat{a}_\theta - \sin\tfrac{3\pi}{2}\,\hat{a}_\phi$$

$$= +\hat{a}_\phi$$

$$\bar{J}_D = \left(+\hat{a}_\phi \cdot \left(-\hat{a}_\theta + 0.693147\,\hat{a}_\phi\right)\right)\hat{a}_x \quad \text{or } +\hat{a}_\phi$$

$$= \left(0 + 0.693147\right)\hat{a}_x$$

$$\underline{\underline{\bar{J}_D = 0.693147\,\hat{a}_x = 0.693147\,\hat{a}_\phi}}$$