2.27 Let $\mathbf{A} = (2z - \sin\phi)\mathbf{a}_\rho + (4\rho + 2\cos\phi)\mathbf{a}_\phi - 3\rho z\mathbf{a}_z$ and $\mathbf{B} = \rho\cos\phi\mathbf{a}_\rho + \sin\phi\mathbf{a}_\phi + \mathbf{a}_z$.

(a) Find the minimum angle between **A** and **B** at $(1, 60°, -1)$.

(b) Determine a unit vector normal to both **A** and **B** at $(1, 90°, 0)$.

a) @ $(1, 60°, -1)$, $\bar{A} = [2(-1) - \sin 60°]\hat{a}_\rho + [4(1) + 2\cos 60°]\hat{a}_\phi - 3(1)(-1)\hat{a}_z$

$= -2.866\hat{a}_\rho + 5\hat{a}_\phi + 3\hat{a}_z$

and $\bar{B} = 1\cos 60°\hat{a}_\rho + \sin 60°\hat{a}_\phi + \hat{a}_z$

$= 0.5\hat{a}_\rho + 0.866\hat{a}_\phi + \hat{a}_z$

Per dot product $\bar{A} \cdot \bar{B} = |\bar{A}||\bar{B}|\cos\theta_{AB}$

$\bar{A} \cdot \bar{B} = -2.866(0.5) + 5(0.866) + 3(1) = 5.8971.$

$|\bar{A}| = \sqrt{(-2.866)^2 + 5^2 + 3^2} = 6.497238$

$|\bar{B}| = \sqrt{(0.5)^2 + 0.866^2 + 1^2} = \sqrt{2} = 1.4142136$

$$\theta_{AB} = \cos^{-1}\left(\frac{5.8971}{6.497\sqrt{2}}\right) = \cos^{-1}(0.641794) = \underline{\underline{50.074°}}$$

b) @ $(1, 90°, 0)$, $\bar{A} = [2(0) - \sin 90°]\hat{a}_\rho + [4(1) + 2\cos 90°]\hat{a}_\phi - 3(1)0\,\hat{a}_z$

$= -\hat{a}_\rho + 4\hat{a}_\phi$

and $\bar{B} = 1\cos 90°\hat{a}_\rho + \sin 90°\hat{a}_\phi + \hat{a}_z = \hat{a}_\phi + \hat{a}_z$

$$\bar{A} \times \bar{B} = \left|\begin{array}{ccc|cc} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z & \hat{a}_\rho & \hat{a}_\phi \\ -1 & 4 & 0 & -1 & 4 \\ 0 & 1 & 1 & 0 & 1 \end{array}\right| = \hat{a}_\rho[4(1) - 0(1)] + \hat{a}_\phi[0 - (-1)(1)] + \hat{a}_z[-1(1) - 4(0)]$$

$= 4\hat{a}_\rho + \hat{a}_\phi - \hat{a}_z$

$|\bar{A} \times \bar{B}| = \sqrt{4^2 + 1^2 + (-1)^2} = \sqrt{18}$

$$\hat{a}_n = \pm\frac{\bar{A} \times \bar{B}}{|\bar{A} \times \bar{B}|} = \pm\frac{4\hat{a}_\rho + \hat{a}_\phi - \hat{a}_z}{\sqrt{18}}$$

$$\underline{\underline{\hat{a}_n = \pm(0.9428\hat{a}_\rho + 0.2357\hat{a}_\phi - 0.2357\hat{a}_z)}}$$