2.10 Express the following vectors in rectangular coordinates:

(a) $\mathbf{A} = \rho \sin\phi\, \mathbf{a}_\rho + \rho \cos\phi\, \mathbf{a}_\phi - 2z\, \mathbf{a}_z$

(b) $\mathbf{B} = 4r \cos\phi\, \mathbf{a}_r + r\, \mathbf{a}_\theta$

a) Per (2.15) & (2.7)

$$A_x = A_\rho \cos\phi - A_\phi \sin\phi$$
$$= \rho \cos\phi \sin\phi - \rho \cos\phi \sin\phi = 0$$

$$A_y = A_\rho \sin\phi + A_\phi \cos\phi$$
$$= \rho \sin^2\phi + \rho \cos^2\phi = \rho = \sqrt{x^2+y^2}$$

$$A_z = A_z = -2z$$

$$\underline{\bar{A}_{cart} = \sqrt{x^2+y^2}\, \hat{a}_y - 2z\, \hat{a}_z}$$

b) Per notes, (2.20), (2.21) & (2.22)

$$B_x = B_r \sin\theta \cos\phi + B_\theta \cos\theta \cos\phi - B_\phi \sin\phi$$
$$= 4r \cos^2\phi \sin\theta + r \cos\theta \cos\phi - 0$$
$$= 4\sqrt{x^2+y^2+z^2}\, \frac{x^2}{x^2+y^2}\, \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}} + \sqrt{x^2+y^2+z^2}\, \frac{z}{\sqrt{x^2+y^2+z^2}}\, \frac{x}{\sqrt{x^2+y^2}}$$
$$= \frac{4x^2 + xz}{\sqrt{x^2+y^2}}$$

$$B_y = B_r \sin\theta \sin\phi + B_\theta \cos\theta \sin\phi + B_\phi \cos\phi$$
$$= 4r \cos\phi \sin\phi \sin\theta + r \cos\theta \sin\phi + 0$$
$$= 4\sqrt{x^2+y^2+z^2}\, \frac{xy}{x^2+y^2}\, \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}} + \sqrt{x^2+y^2+z^2}\, \frac{z}{\sqrt{x^2+y^2+z^2}}\, \frac{y}{\sqrt{x^2+y^2}}$$
$$= \frac{4xy + yz}{\sqrt{x^2+y^2}}$$

b) cont.

$$B_z = B_r \cos\theta - B_\theta \sin\theta$$

$$= 4r\cos\phi\cos\theta - r\sin\theta$$

$$= 4\sqrt{x^2+y^2+z^2}\,\frac{x}{\sqrt{x^2+y^2}}\,\frac{z}{\sqrt{x^2+y^2+z^2}} - \sqrt{x^2+y^2+z^2}\,\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$$

$$= \frac{4xz}{\sqrt{x^2+y^2}} - \sqrt{x^2+y^2} = \frac{4xz - x^2 - y^2}{\sqrt{x^2+y^2}}$$

$$\bar{B}_{cart} = \frac{1}{\sqrt{x^2+y^2}}\left[(4x^2+xz)\hat{a}_x + (4xy+yz)\hat{a}_y + (4xz-x^2-y^2)\hat{a}_z\right]$$

$$= \frac{1}{\sqrt{x^2+y^2}}\left[x(4x+z)\hat{a}_x + y(4x+z)\hat{a}_y + (4xz-x^2-y^2)\hat{a}_z\right]$$