PE1.5 Let $\overline{E} = 3\hat{a}_y + 4\hat{a}_z$ and $\overline{F} = 4\hat{a}_x - 10\hat{a}_y + 5\hat{a}_z$,

(a) Find the component of $\overline{E}$ along $\overline{F}$.

(b) Determine a unit vector perpendicular to both $\overline{E}$ and $\overline{F}$.

a) $\overline{E}_F = (\overline{E} \cdot \hat{a}_F)\hat{a}_F$ where

$$\hat{a}_F = \frac{\overline{F}}{|\overline{F}|} = \frac{4\hat{a}_x - 10\hat{a}_y + 5\hat{a}_z}{\sqrt{4^2 + (-10)^2 + 5^2}} = \frac{4\hat{a}_x - 10\hat{a}_y + 5\hat{a}_z}{\sqrt{141}}$$

$$\overline{E}_F = \frac{\big(0(4) + 3(-10) + 4(5)\big)\big(4\hat{a}_x - 10\hat{a}_y + 5\hat{a}_z\big)}{(\sqrt{141})^2}$$

$$= \frac{-40\hat{a}_x + 100\hat{a}_y - 50\hat{a}_z}{141}$$

$$\underline{\overline{E}_F = -0.2837\hat{a}_x + 0.7092\hat{a}_y - 0.3546\hat{a}_z}$$

b)
$$\overline{E} \times \overline{F} = \left| \begin{array}{ccc|cc} \hat{a}_x & \hat{a}_y & \hat{a}_z & \hat{a}_x & \hat{a}_y \\ 0 & 3 & 4 & 0 & 3 \\ 4 & -10 & 5 & 4 & -10 \end{array} \right|$$

$$= \big(3(5) - (4)(-10)\big)\hat{a}_x + \big(4(4) - 0(5)\big)\hat{a}_y + \big(0(-10) - 3(4)\big)\hat{a}_z$$

$$= 55\hat{a}_x + 16\hat{a}_y - 12\hat{a}_z$$

$$\pm\frac{\overline{E} \times \overline{F}}{|\overline{E} \times \overline{F}|} = \pm\frac{55\hat{a}_x + 16\hat{a}_y - 12\hat{a}_z}{\sqrt{(55)^2 + 16^2 + (-12)^2}} = \underline{\underline{\pm\big(0.9398\hat{a}_x + 0.2734\hat{a}_y - 0.205\hat{a}_z\big)}}$$