

1.30 A vector field is given by $\mathbf{H} = 10yz^2\mathbf{a}_x - 8xyza_y + 12y^2\mathbf{a}_z$

(a) Evaluate \mathbf{H} at $P(-1, 2, 4)$

(b) Find the component of \mathbf{H} along $\mathbf{a}_x - \mathbf{a}_y$ at P .

$$a) \quad \bar{H}_P = 10(2)4^2\hat{a}_x - 8(-1)2(4)\hat{a}_y + 12(2)^2\hat{a}_z$$

$$\bar{H}_P = \underline{\underline{320\hat{a}_x + 64\hat{a}_y + 48\hat{a}_z}}$$

b) Find unit vector for $\bar{A} = \hat{a}_x - \hat{a}_y$

$$\hat{a}_A = \frac{\bar{A}}{|\bar{A}|} = \frac{\hat{a}_x - \hat{a}_y}{\sqrt{1^2 + (-1)^2}} = \frac{1}{\sqrt{2}}\hat{a}_x - \frac{1}{\sqrt{2}}\hat{a}_y$$

$$\bar{H}_{A,P} = (\bar{H}_P \cdot \hat{a}_A) \hat{a}_A$$

$$= \left[(320\hat{a}_x + 64\hat{a}_y + 48\hat{a}_z) \cdot \left(\frac{\hat{a}_x}{\sqrt{2}} - \frac{\hat{a}_y}{\sqrt{2}} \right) \right] \left(\frac{\hat{a}_x}{\sqrt{2}} - \frac{\hat{a}_y}{\sqrt{2}} \right)$$

$$= \left[\frac{320}{\sqrt{2}} - \frac{64}{\sqrt{2}} \right] \left(\frac{\hat{a}_x}{\sqrt{2}} - \frac{\hat{a}_y}{\sqrt{2}} \right)$$

$$= \frac{256}{\sqrt{2}} \left(\frac{\hat{a}_x}{\sqrt{2}} - \frac{\hat{a}_y}{\sqrt{2}} \right)$$

$$\bar{H}_{A,P} = \underline{\underline{128\hat{a}_x - 128\hat{a}_y}}$$