1.23 Given vectors $\mathbf{T} = 2\mathbf{a}_x - 6\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{S} = \mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$, find (a) the scalar projection of **T** on **S**, (b) the vector projection of **S** on **T**, (c) the smaller angle between **T** and **S**.

a) $\overline{T}\cdot\hat{a}_S = \dfrac{\overline{T}\cdot\overline{S}}{|\overline{S}|} = \dfrac{(2\hat{a}_x - 6\hat{a}_y + 3\hat{a}_z)\cdot(\hat{a}_x + 2\hat{a}_y + \hat{a}_z)}{\sqrt{1^2+(2)^2+1^2}}$

$\overline{T}\cdot\hat{a}_S = \dfrac{2(1)-6(2)+3(1)}{\sqrt{6}} = -2.85774$

b) $\overline{S}_T = (\overline{S}\cdot\hat{a}_T)\hat{a}_T = \dfrac{(\overline{S}\cdot\overline{T})\overline{T}}{|\overline{T}|^2} = \dfrac{(2(1)-6(2)+3(1))\overline{T}}{(2^2+(-6)^2+3^2)}$

$= \dfrac{-7}{49}(2\hat{a}_x - 6\hat{a}_y + 3\hat{a}_z)$

$\overline{S}_T = -0.2857\hat{a}_x + 0.8574\hat{a}_y - 0.42857\hat{a}_z$

c) $\overline{T}\cdot\overline{S} = 2(1)-6(2)+3(1) = -7$

$|\overline{T}| = \sqrt{\overline{T}\cdot\overline{T}} = \sqrt{2^2+(-6)^2+3^2} = 7$

$|\overline{S}| = \sqrt{\overline{S}\cdot\overline{S}} = \sqrt{1^2+2^2+1^2} = \sqrt{6}$

$\overline{T}\cdot\overline{S} = |\overline{T}||\overline{S}|\cos\theta_{TS}$

$\cos\theta_{TS} = \dfrac{-7}{7\sqrt{6}} = -0.40825$

$\theta_{TS} = \cos^{-1}(-0.40825)$

$\theta_{TS} = 114.095^\circ$