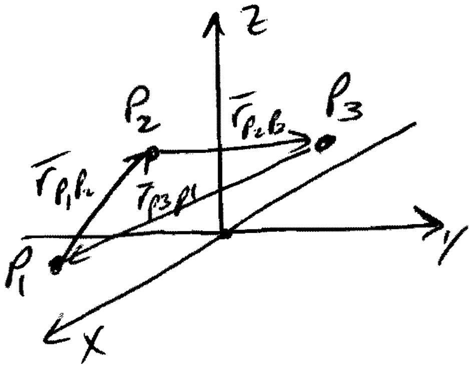
1.16 A right angle triangle has its corners located at $P_1(5, -3, 1)$, $P_2(1, -2, 4)$, and $P_3(3, 3, 5)$. (a) Which corner is a right angle? (b) Calculate the area of the triangle.

- Assume units of meters.



position vectors:

$$\bar{r}_{P_1} = 5\hat{a}_x - 3\hat{a}_y + \hat{a}_z \text{ (m)}$$

$$\bar{r}_{P_2} = \hat{a}_x - 2\hat{a}_y + 4\hat{a}_z \text{ (m)}$$

$$\bar{r}_{P_3} = 3\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z \text{ (m)}$$

distance vectors:

$$\bar{r}_{P_1P_2} = \bar{r}_{P_2} - \bar{r}_{P_1} = (1-5)\hat{a}_x + (-2+3)\hat{a}_y + (4-1)\hat{a}_z$$
$$= -4\hat{a}_x + \hat{a}_y + 3\hat{a}_z \text{ (m)}$$

$$\bar{r}_{P_2P_3} = \bar{r}_{P_3} - \bar{r}_{P_2} = (3-1)\hat{a}_x + (3+2)\hat{a}_y + (5-4)\hat{a}_z$$
$$= 2\hat{a}_x + 5\hat{a}_y + \hat{a}_z \text{ (m)}$$

$$\bar{r}_{P_3P_1} = \bar{r}_{P_1} - \bar{r}_{P_3} = (5-3)\hat{a}_x + (-3-3)\hat{a}_y + (+1-5)\hat{a}_z$$
$$= 2\hat{a}_x - 6\hat{a}_y - 4\hat{a}_z \text{ (m)}$$

a) For a right angle, the dot product of the direction vectors must be zero (i.e., $\cos\theta = \cos 90° = 0$)

check vertex $P_1 \rightarrow \bar{r}_{P_1P_2} \cdot \bar{r}_{P_3P_1} = -4(2) + 1(-6) + (3)(-4) = -26 \neq 0$

check vertex $P_2 \rightarrow \bar{r}_{P_1P_2} \cdot \bar{r}_{P_2P_3} = (-4)(2) + 1(5) + 1(3) = \underline{0}$

<u>Vertex P2 is the right angle corner</u>

b) Area $= \frac{1}{2} \left| \text{side}_i \times \text{side}_j \right|$

$$\bar{r}_{P_1P_2} \times \bar{r}_{P_2P_3} = \begin{vmatrix} a_x & \hat{a}_y & \hat{a}_z \\ -4 & 1 & 3 \\ 2 & 5 & 1 \end{vmatrix} \begin{matrix} \hat{a}_x & \hat{a}_y \\ -4 & 1 \\ 2 & 5 \end{matrix} = \begin{matrix} \hat{a}_x[1(1) - 3(5)] + \\ \hat{a}_y[3(2) - (-4)(1)] + \\ \hat{a}_z[-4(5) - 2(1)] \end{matrix}$$

$$= -14\hat{a}_x + 10\hat{a}_y - 22\hat{a}_z$$

$$\text{Area} = \frac{1}{2}\sqrt{(-14)^2 + 10^2 + (-22)^2} = \frac{\sqrt{780}}{2} = \underline{13.964 \text{ m}^2}$$