1.13 Determine the dot product, cross product, and angle between

$$\mathbf{P} = 2\mathbf{a}_x - 6\mathbf{a}_y + 5\mathbf{a}_z \qquad \text{and} \qquad \mathbf{Q} = 3\mathbf{a}_y + \mathbf{a}_z$$

$$\bar{P} \cdot \bar{Q} = (2\hat{a}_x - 6\hat{a}_y + 5\hat{a}_z) \cdot (3\hat{a}_y + \hat{a}_z)$$

$$= 2(0) + (-6)(3) + 5(1)$$

$$\underline{\underline{\bar{P} \cdot \bar{Q} = -13}}$$

$$\bar{P} \times \bar{Q} = \left| \begin{array}{ccc|cc} \hat{a}_x & \hat{a}_y & \hat{a}_z & \hat{a}_x & \hat{a}_y \\ 2 & -6 & 5 & 2 & -6 \\ 0 & 3 & 1 & 0 & 3 \end{array} \right| = \begin{array}{l} \hat{a}_x[-6(1) - 3(5)] \\ +\hat{a}_y[5(0) - 2(1)] \\ +\hat{a}_z[2(3) - (-6)0] \end{array}$$

$$\underline{\underline{\bar{P} \times \bar{Q} = -21\hat{a}_x - 2\hat{a}_y + 6\hat{a}_z}}$$

By definition $\bar{P} \cdot \bar{Q} = |\bar{P}||\bar{Q}| \cos\theta_{PQ}$

$$\theta_{PQ} = \cos^{-1}\left[\frac{\bar{P} \cdot \bar{Q}}{|\bar{P}||\bar{Q}|}\right]$$

$$|\bar{P}| = \sqrt{\bar{P} \cdot \bar{P}} = \sqrt{(2\hat{a}_x - 6\hat{a}_y + 5\hat{a}_z) \cdot (2\hat{a}_x - 6\hat{a}_y + 5\hat{a}_z)}$$

$$= \sqrt{2^2 + (-6)^2 + 5^2} = \sqrt{65}$$

$$|\bar{Q}| = \sqrt{\bar{Q} \cdot \bar{Q}} = \sqrt{(3\hat{a}_y + \hat{a}_z) \cdot (3\hat{a}_y + \hat{a}_z)} = \sqrt{3^2 + 1^2} = \sqrt{10}$$

$$\theta_{PQ} = \cos^{-1}\left[\frac{-13}{\sqrt{65}\sqrt{10}}\right] = \cos^{-1}(-0.5099)$$

$$\underline{\underline{\theta_{PQ} = 120.657^\circ}}$$