1.11 Given that

$$\mathbf{P} = 2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z$$
$$\mathbf{Q} = 4\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z$$
$$\mathbf{R} = -\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z$$

find: (c) $\mathbf{Q} \times \mathbf{P} \cdot \mathbf{R}$, (d) $(\mathbf{P} \times \mathbf{Q}) \cdot (\mathbf{Q} \times \mathbf{R})$, (f) $\cos \theta_{PR}$

c)

$$\bar{Q} \times \bar{P} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 4 & 3 & 2 \\ 2 & -1 & -2 \end{vmatrix} \quad (1.22a)$$

$$= \hat{a}_x[3(-2) - 2(-1)] + \hat{a}_y[2(2) - 4(-2)] + \hat{a}_z[4(-1) - 3(2)]$$

$$= -4\hat{a}_x + 12\hat{a}_y - 10\hat{a}_z$$

$$(\bar{Q} \times \bar{P}) \cdot \bar{R} = (-4\hat{a}_x + 12\hat{a}_y - 10\hat{a}_z) \cdot (-\hat{a}_x + \hat{a}_y + 2\hat{a}_z)$$

$$= -4(-1) + 12(1) - 10(2)$$

$$\underline{(\bar{Q} \times \bar{P}) \cdot \bar{R} = -4}$$

d) use (1.23b), $\bar{P} \times \bar{Q} = -\bar{Q} \times \bar{P} = +4\hat{a}_x - 12\hat{a}_y + 10\hat{a}_z$

$$\bar{Q} \times \bar{R} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 4 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix}$$

$$= \hat{a}_x[3(2) - 2(1)] + \hat{a}_y[2(-1) - 4(2)] + \hat{a}_z[4(1) - 3(-1)]$$

$$= 4\hat{a}_x - 10\hat{a}_y + 7\hat{a}_z$$

$$(\bar{P} \times \bar{Q}) \cdot (\bar{Q} \times \bar{R}) = (4\hat{a}_x - 12\hat{a}_y + 10\hat{a}_z) \cdot (4\hat{a}_x - 10\hat{a}_y + 7\hat{a}_z)$$

$$= 4(4) + (-12)(-10) + 10(7)$$

$$\underline{(\bar{P} \times \bar{Q}) \cdot (\bar{Q} \times \bar{R}) = 206}$$

f) $\cos\theta_{PR}$

$$\bar{P}\cdot\bar{R} = |\bar{P}||\bar{R}|\cos\theta_{PR}$$

$$\cos\theta_{PR} = \frac{\bar{P}\cdot\bar{R}}{|\bar{P}||\bar{R}|} = \frac{(2\hat{a}_x - \hat{a}_y - 2\hat{a}_z)\cdot(-\hat{a}_x + \hat{a}_y + 2\hat{a}_z)}{\sqrt{2^2+(-1)^2+(-2)^2}\sqrt{(-1)^2+(1)^2+(2)^2}}$$

$$= \frac{2(-1)+(-1)(1)+(-2)(2)}{\sqrt{9}\sqrt{6}} = \frac{-2-1-4}{3\sqrt{6}} = \frac{-7}{3\sqrt{6}}$$

$$\underline{\cos\theta_{PR} = -0.95258}$$

$$\Rightarrow \theta_{PR} = 162.28^\circ$$