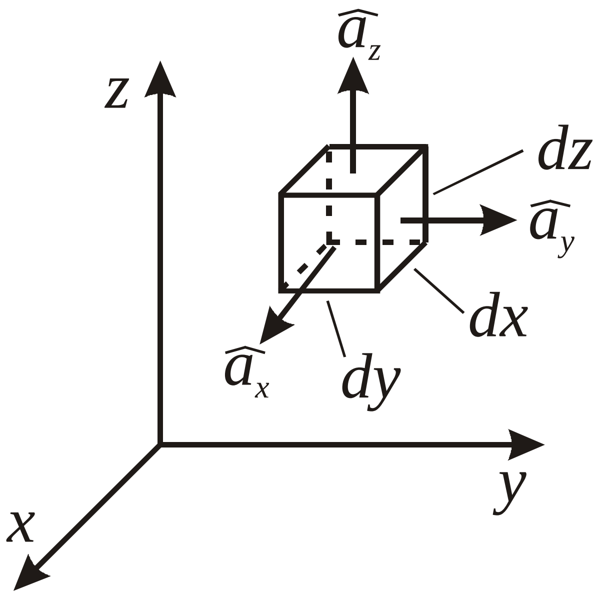


Rectangular/Cartesian Coordinates (x,y,z)

$$d\overline{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z \quad (\text{m})$$

$$d\overline{s}_x = dy\,dz\,\hat{a}_x \qquad d\overline{s}_y = dx\,dz\,\hat{a}_y \qquad d\overline{s}_z = dx\,dy\,\hat{a}_z \quad (\text{m}^2)$$

$$ds_x = dy\,dz \qquad ds_y = dx\,dz \qquad ds_z = dx\,dy \quad (\text{m}^2)$$

$$dv = dx\,dy\,dz \quad (\text{m}^3)$$

$$\overline{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z \quad (\text{m})$$

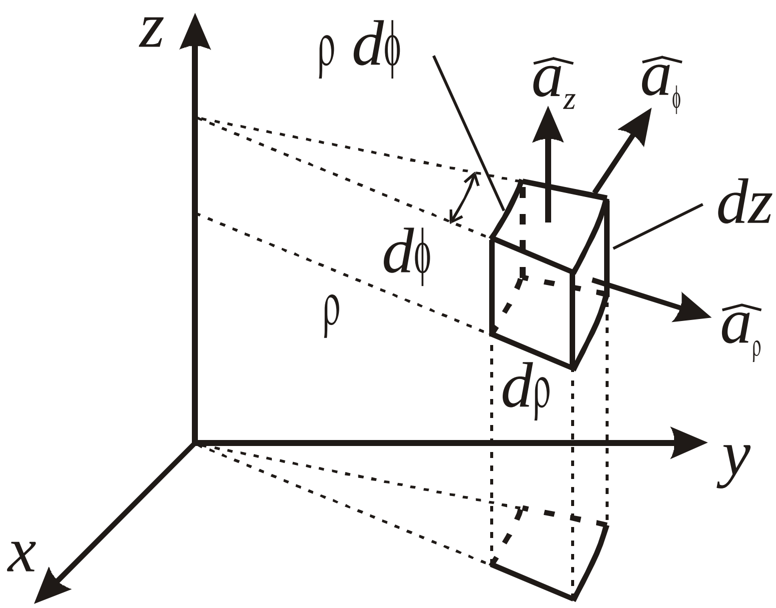
$$\overline{\nabla}\Phi = \hat{a}_x \frac{\partial \Phi}{\partial x} + \hat{a}_y \frac{\partial \Phi}{\partial y} + \hat{a}_z \frac{\partial \Phi}{\partial z} \qquad \left(\frac{\text{units of } \Phi}{\text{m}}\right)$$

$$\overline{\nabla} \bullet \overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad \left(\frac{\text{units of } A}{\text{m}}\right)$$

$$\overline{\nabla} \times \overline{A} = \hat{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\text{with} \left(\frac{\text{units of } A}{\text{m}}\right)$$

$$\nabla^2\Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} \qquad \left(\frac{\text{units of } \Phi}{\text{m}^2}\right)$$



Cylindrical Coordinates (ρ,ϕ,z)

$$d\bar{l} = d\rho\,\hat{a}_\rho + \rho d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$$

$$d\bar{s}_\rho = \rho\,d\phi\,dz\,\hat{a}_\rho \qquad d\bar{s}_\phi = d\rho\,dz\,\hat{a}_\phi \qquad d\bar{s}_z = \rho\,d\rho\,d\phi\,\hat{a}_z$$

$$ds_\rho = \rho\,d\phi\,dz \qquad ds_\phi = d\rho\,dz \qquad ds_z = \rho\,d\rho\,d\phi$$

$$dv = \rho\,d\rho\,d\phi\,dz$$

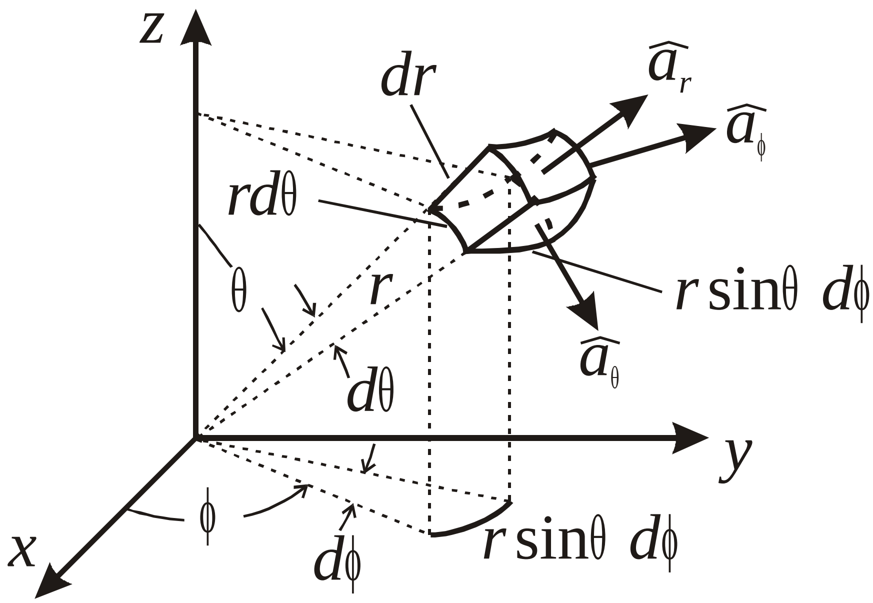
$$\bar{r} = \rho\,\hat{a}_\rho + z\,\hat{a}_z$$ (Note: $\hat{a}_\rho$ has ϕ dependence)

$$\bar{\nabla}\Phi = \hat{a}_\rho \frac{\partial \Phi}{\partial \rho} + \hat{a}_\phi \frac{1}{\rho}\frac{\partial \Phi}{\partial \phi} + \hat{a}_z \frac{\partial \Phi}{\partial z}$$

$$\bar{\nabla} \bullet \bar{A} = \frac{1}{\rho}\frac{\partial\left(\rho A_\rho\right)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\bar{\nabla} \times \bar{A} = \hat{a}_\rho \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{a}_\phi \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \hat{a}_z \left(\frac{1}{\rho}\frac{\partial\left(\rho A_\phi\right)}{\partial \rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial \phi}\right)$$

$$\nabla^2\Phi = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \Phi}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \Phi}{\partial \phi^2} + \frac{\partial^2 \Phi}{\partial z^2}$$



Spherical Coordinates (r,θ,ϕ)

$$d\bar{l} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$$

$$d\bar{s}_r = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r \qquad ds_r = r^2\sin\theta\,d\theta\,d\phi$$

$$d\bar{s}_\theta = r\sin\theta\,dr\,d\phi\,\hat{a}_\theta \qquad ds_\theta = r\sin\theta\,dr\,d\phi$$

$$d\bar{s}_\phi = r\,dr\,d\theta\,\hat{a}_\phi \qquad ds_\phi = r\,dr\,d\theta$$

$$dv = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$\bar{r} = r\,\hat{a}_r$ (Note: $\hat{a}_r$ has both θ and ϕ dependence)

$$\bar{\nabla}\Phi = \hat{a}_r\frac{\partial\Phi}{\partial r} + \hat{a}_\theta\frac{1}{r}\frac{\partial\Phi}{\partial\theta} + \hat{a}_\phi\frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}$$

$$\bar{\nabla}\bullet\bar{A} = \frac{1}{r^2}\frac{\partial\left(r^2A_r\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial\left(\sin\theta\,A_\theta\right)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\bar{\nabla}\times\bar{A} = \hat{a}_r\left\{\frac{1}{r\sin\theta}\left[\frac{\partial\left(\sin\theta\,A_\phi\right)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\right\} + \hat{a}_\theta\left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial\left(rA_\phi\right)}{\partial r}\right]$$

$$+\ \hat{a}_\phi\left\{\frac{1}{r}\left[\frac{\partial\left(rA_\theta\right)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\right\}$$

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$$