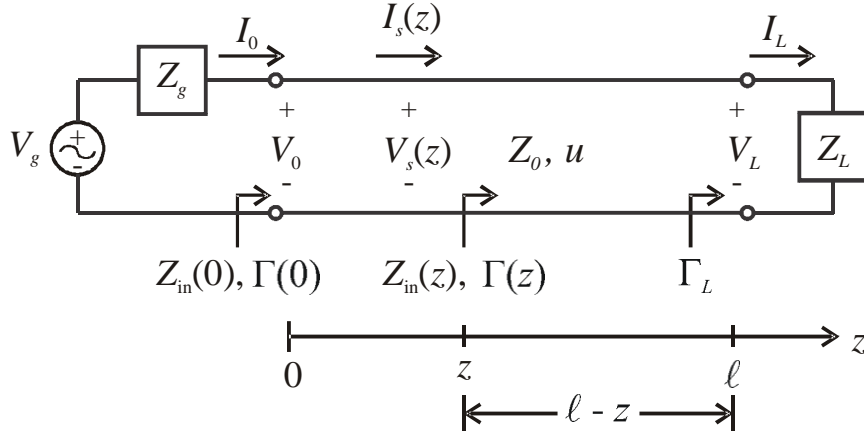


Lossless Transmission Lines



Phase velocity: $u = f\lambda = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$ (m/s)

where f is the frequency (Hz), ω is the radian frequency (rad/s), and L and C are the per-unit-length inductance and capacitance.

Phase constant: $\beta = \frac{\omega}{u} = \omega\sqrt{LC} = \frac{2\pi}{\lambda}$ (rad/m)

Wavelength: $\lambda = \frac{u}{f} = \frac{2\pi}{\beta}$ (m)

Characteristic impedance: $Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \sqrt{\frac{L}{C}}$ (Ω)

Phasor voltage: $V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$ (V)
 $= V_0^+ e^{-j\beta z} [1 + \Gamma(z)]$ (V)

$I_s(z) = I_0^+ e^{-j\beta z} + I_0^- e^{+j\beta z}$ (A)

Phasor current: $= \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z}$ (A)

$= \frac{V_0^+}{Z_0} e^{-j\beta z} [1 - \Gamma(z)]$ (A)

where $V_0^+ = |V_0^+| \angle \theta^+$, $V_0^- = |V_0^-| \angle \theta^-$, I_0^+ , and I_0^- are complex coefficients representing the forward and backward traveling voltages and currents.

Forward and backward traveling voltages and currents:

Note that, at the input ($z = 0$), $V_s(0) = V_0 = V_0^+ + V_0^-$ and $I_s(0) = I_0 = I_0^+ + I_0^- = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$

$V_0^+ = 0.5(V_0 + I_0 Z_0) = \frac{V_0}{1 + \Gamma(0)} = 0.5(V_L + I_L Z_0) e^{+j\beta l}$ and $I_0^+ = \frac{V_0^+}{Z_0}$

$V_0^- = 0.5(V_0 - I_0 Z_0) = V_0 - V_0^+ = 0.5(V_L - I_L Z_0) e^{-j\beta l}$ and $I_0^- = \frac{-V_0^-}{Z_0}$.

Time-domain voltage: $V(z,t) = |V_0^+| \cos(\omega t - \beta z + \theta^+) + |V_0^-| \cos(\omega t + \beta z + \theta^-)$ (V)

Time-domain current: $I(z,t) = \frac{|V_0^+|}{Z_0} \cos(\omega t - \beta z + \theta^+) - \frac{|V_0^-|}{Z_0} \cos(\omega t + \beta z + \theta^-)$ (A)

Reflection Coefficient: $\Gamma(z) = \frac{V_{\text{ref}}(z)}{V_{\text{inc}}(z)} = -\frac{I_{\text{ref}}(z)}{I_{\text{inc}}(z)} = \frac{Z_{\text{in}}(z) - Z_0}{Z_{\text{in}}(z) + Z_0} = \Gamma_L e^{-j2\beta(l-z)}$,

of particular interest are: load $\Gamma_L = \frac{V_0^-}{V_0^+} e^{+j2\beta l} = \Gamma(0) e^{+j2\beta l} = \frac{Z_L - Z_0}{Z_L + Z_0}$ and

input $\Gamma(0) = \frac{Z_{\text{in}}(0) - Z_0}{Z_{\text{in}}(0) + Z_0} = \Gamma_L e^{-j2\beta l}$ reflection coefficients.

Notes: $|\Gamma(z)|$ is constant and $0 \leq |\Gamma(z)| \leq 1$ for passive loads.

Input impedance: $Z_{\text{in}}(z) = \frac{V(z)}{I(z)} = Z_0 \left[\frac{Z_L + jZ_0 \tan[\beta(\ell - z)]}{Z_0 + jZ_L \tan[\beta(\ell - z)]} \right] = Z_0 \left[\frac{1 + \Gamma(z)}{1 - \Gamma(z)} \right]$ for $0 \leq z \leq l$

at the input $z = 0$: $Z_{\text{in}}(0) = \frac{V(0)}{I(0)} = \frac{V_0}{I_0} = Z_0 \left[\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \right] = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)} \right] = Z_0 \left[\frac{1 + \Gamma(0)}{1 - \Gamma(0)} \right]$

at the load $z = l$: $Z_{\text{in}}(l) = Z_L = \frac{V_L}{I_L} = Z_0 \left[\frac{1 + \Gamma_L}{1 - \Gamma_L} \right]$

Standing wave ratio: $SWR = S = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{I_{\text{max}}}{I_{\text{min}}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$ and $1 \leq S < \infty$ for passive loads.

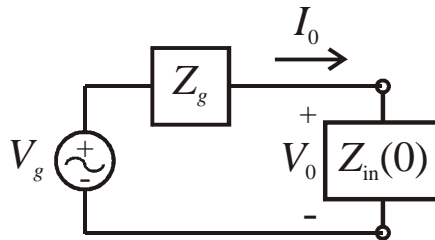
$$P_{\text{ave}}(z) = 0.5 \text{Re}\{V(z)I^*(z)\} = 0.5 \frac{|V_0^+|^2}{Z_0} [1 - |\Gamma(z)|^2] \quad (\text{W})$$

Input Power: $= P_{\text{ave}}(0) = 0.5 \text{Re}\{V_0 I_0^*\} = 0.5 \frac{|V_0^+|^2}{Z_0} [1 - |\Gamma(0)|^2]$ (W)

$$= P_{\text{ave}}(l) = 0.5 \text{Re}\{V_L I_L^*\} \quad (\text{W})$$

Note: Time-average real power is constant on a lossless transmission line.

The input phasor voltage $V_s(0) = V_0$ and current $I_s(0) = I_0$ can be found using the equivalent circuit-



$$V_0 = V_g \left(\frac{Z_{\text{in}}(0)}{Z_g + Z_{\text{in}}(0)} \right) \text{ and } I_0 = \frac{V_g}{Z_g + Z_{\text{in}}(0)}$$