Rectangular/Cartesian Coordinates (x, y, z)

$$d\overline{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

$$d\overline{s}_x = dy\,dz\,\hat{a}_x \qquad d\overline{s}_y = dx\,dz\,\hat{a}_y \qquad d\overline{s}_z = dx\,dy\,\hat{a}_z$$

$$dv = dx\,dy\,dz$$

$$\overline{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$$

$$\overline{\nabla}\Phi = \hat{a}_x\frac{\partial\Phi}{\partial x} + \hat{a}_y\frac{\partial\Phi}{\partial y} + \hat{a}_z\frac{\partial\Phi}{\partial z}$$

$$\overline{\nabla}\bullet\overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\overline{\nabla}\times\overline{A} = \hat{a}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{a}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{a}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}$$

Cylindrical Coordinates (ρ, ϕ, z)

$$d\overline{l} = d\rho\,\hat{a}_\rho + \rho d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$$

$$d\overline{s}_\rho = \rho\,d\phi\,dz\,\hat{a}_\rho \qquad d\overline{s}_\phi = d\rho\,dz\,\hat{a}_\phi \qquad d\overline{s}_z = \rho\,d\rho\,d\phi\,\hat{a}_z$$

$$dv = \rho\,d\rho\,d\phi\,dz$$

$\overline{r} = \rho\,\hat{a}_\rho + z\,\hat{a}_z$ (Note: $\hat{a}_\rho$ has ϕ dependence)

$$\overline{\nabla}\Phi = \hat{a}_\rho\frac{\partial\Phi}{\partial\rho} + \hat{a}_\phi\frac{1}{\rho}\frac{\partial\Phi}{\partial\phi} + \hat{a}_z\frac{\partial\Phi}{\partial z}$$

$$\overline{\nabla}\bullet\overline{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\overline{\nabla}\times\overline{A} = \hat{a}_\rho\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{a}_\phi\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right) + \hat{a}_z\left(\frac{1}{\rho}\frac{\partial(\rho A_\phi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial\phi}\right)$$

$$\nabla^2\Phi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2}$$

Spherical Coordinates (r, θ, ϕ)

$$d\overline{l} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$$

$$d\overline{s}_r = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r$$

$$d\overline{s}_\theta = r\sin\theta\,dr\,d\phi\,\hat{a}_\theta$$

$$d\overline{s}_\phi = r\,dr\,d\theta\,\hat{a}_\phi$$

$$dv = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$\overline{r} = r\,\hat{a}_r$ (Note: $\hat{a}_r$ has both θ and ϕ dependence)

$$\overline{\nabla}\Phi = \hat{a}_r\frac{\partial\Phi}{\partial r} + \hat{a}_\theta\frac{1}{r}\frac{\partial\Phi}{\partial\theta} + \hat{a}_\phi\frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}$$

$$\overline{\nabla}\bullet\overline{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\,A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\overline{\nabla}\times\overline{A} = \hat{a}_r\left\{\frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta\,A_\phi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\right\} + \hat{a}_\theta\left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial(rA_\phi)}{\partial r}\right] + \hat{a}_\phi\left\{\frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\right\}$$

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$$

Cartesian (x, y, z) ⇔ Cylindrical (ρ, ϕ, z)

Point/variable conversions:

$$\rho = \sqrt{x^2+y^2} \qquad x = \rho\cos\phi \qquad \cos\phi = \frac{x}{\sqrt{x^2+y^2}}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \qquad y = \rho\sin\phi \qquad \sin\phi = \frac{y}{\sqrt{x^2+y^2}}$$

$$z = z \qquad z = z$$

Vector conversions:

$$\overline{A} = \hat{a}_\rho A_\rho + \hat{a}_\phi A_\phi + A_z\hat{a}_z = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

$$\hat{a}_x = \cos\phi\,\hat{a}_\rho - \sin\phi\,\hat{a}_\phi \qquad \hat{a}_\rho = \cos\phi\,\hat{a}_x + \sin\phi\,\hat{a}_y$$

$$\hat{a}_y = \sin\phi\,\hat{a}_\rho + \cos\phi\,\hat{a}_\phi \qquad \hat{a}_\phi = -\sin\phi\,\hat{a}_x + \cos\phi\,\hat{a}_y$$

$$\hat{a}_z = \hat{a}_z$$

$$A_\rho = A_x\cos\phi + A_y\sin\phi \qquad A_x = A_\rho\cos\phi - A_\phi\sin\phi$$

$$A_\phi = -A_x\sin\phi + A_y\cos\phi \qquad A_y = A_\rho\sin\phi + A_\phi\cos\phi$$

$$A_z = A_z \qquad A_z = A_z$$

Dot Products:

$$\hat{a}_x\bullet\hat{a}_\rho = \cos\phi \qquad \hat{a}_y\bullet\hat{a}_\rho = \sin\phi \qquad \hat{a}_z\bullet\hat{a}_\rho = 0$$

$$\hat{a}_x\bullet\hat{a}_\phi = -\sin\phi \qquad \hat{a}\bullet\hat{a}_\phi = \cos\phi \qquad \hat{a}_z\bullet\hat{a}_\phi = 0$$

$$\hat{a}_x\bullet\hat{a}_z = 0 \qquad \hat{a}_y\bullet\hat{a}_z = 0 \qquad \hat{a}_z\bullet\hat{a}_z = 1$$

Cartesian (x, y, z) ⇔ Spherical (r, θ, ϕ)

Point/variable conversions:

$$r = \sqrt{x^2+y^2+z^2} \qquad x = r\sin\theta\cos\phi \qquad \cos\phi = \frac{x}{\sqrt{x^2+y^2}} \qquad \cos\theta = \frac{\mathrm{z}}{\sqrt{x^2+y^2+z^2}}$$

$$\phi = \tan^{-1}\frac{y}{x} \qquad y = r\sin\theta\sin\phi$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{\mathrm{z}} \qquad z = r\cos\theta \qquad \sin\phi = \frac{y}{\sqrt{x^2+y^2}} \qquad \sin\theta = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$$

Vector conversions:

$$\overline{A} = \hat{a}_rA_r + A_\theta\hat{a}_\theta + \hat{a}_\phi A_\phi = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

$$\hat{a}_r = \sin\theta\cos\phi\,\hat{a}_x + \sin\theta\sin\phi\,\hat{a}_y + \cos\theta\,\hat{a}_z \qquad \hat{a}_x = \sin\theta\cos\phi\,\hat{a}_r + \cos\theta\cos\phi\,\hat{a}_\theta - \sin\phi\,\hat{a}_\phi$$

$$\hat{a}_\theta = \cos\theta\cos\phi\,\hat{a}_x + \cos\theta\sin\phi\,\hat{a}_y - \sin\theta\,\hat{a}_z \qquad \hat{a}_y = \sin\theta\sin\phi\,\hat{a}_r + \cos\theta\sin\phi\,\hat{a}_\theta + \cos\phi\,\hat{a}_\phi$$

$$\hat{a}_\phi = -\sin\phi\,\hat{a}_x + \cos\phi\,\hat{a}_y \qquad \hat{a}_z = \cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta$$

$$A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta \qquad A_x = A_r\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$$

$$A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta \qquad A_y = A_r\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$$

$$A_\phi = -A_x\sin\phi + A_y\cos\phi \qquad A_z = A_r\cos\theta - A_\theta\sin\theta$$

Dot Products:

$$\hat{a}_r\bullet\hat{a}_x = \sin\theta\cos\phi \qquad \hat{a}_r\bullet\hat{a}_y = \sin\theta\sin\phi \qquad \hat{a}_r\bullet\hat{a}_z = \cos\theta$$

$$\hat{a}_\theta\bullet\hat{a}_x = \cos\theta\cos\phi \qquad \hat{a}_\theta\bullet\hat{a}_y = \cos\theta\sin\phi \qquad \hat{a}_\theta\bullet\hat{a}_z = -\sin\theta$$

$$\hat{a}_\phi\bullet\hat{a}_x = -\sin\phi \qquad \hat{a}_\phi\bullet\hat{a}_y = \cos\phi \qquad \hat{a}_\phi\bullet\hat{a}_z = 0$$