

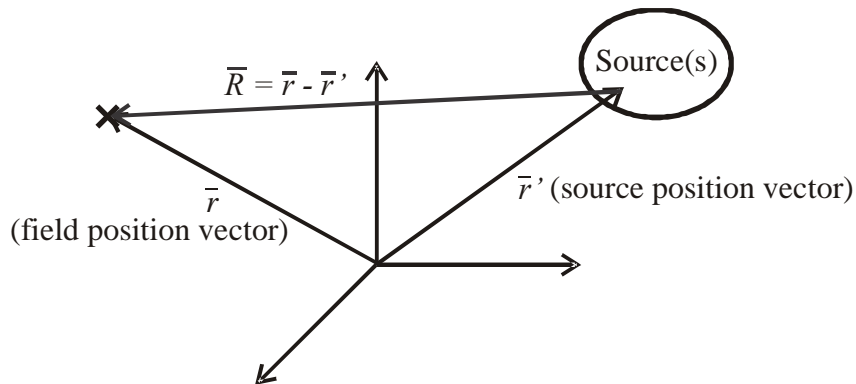
Chapter 1 Vector Algebra

Scalars- magnitude Vectors- direction & magnitude

Position vectors- point from origin to a field point (\vec{r}) or to a source point (\vec{r}')

Distance vectors- usually point from a source point to a field point $\vec{R} = \vec{r} - \vec{r}'$

The distance between the source and the field point is $R = |\vec{R}| = |\vec{r} - \vec{r}'|$



Scalar/dot product: $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos(\theta_{AB}) = \vec{B} \cdot \vec{A}$

Vector/cross product: $\vec{A} \times \vec{B} = \hat{a}_n |\vec{A}| |\vec{B}| \sin(\theta_{AB})$ where \hat{a}_n determined by RHR. Also, $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$.

Cross product hint: Remember order of (x, y, z), (ρ, φ, z), and (r, θ, φ)

Unit vector $\hat{a}_A = \frac{\vec{A}}{|\vec{A}|}$ where the vector magnitude is $|\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}} = \sqrt{|\vec{A}|^2}$.

Scalar projection of vector \vec{A} onto \vec{B} is $A_B = \vec{A} \cdot \hat{a}_B$. The vector projection of vector \vec{A} onto \vec{B} is $\vec{A}_B = \hat{a}_B (\vec{A} \cdot \hat{a}_B)$.

Chapter 2 Coordinate Systems and Transformation & Chapter 3 Vector Calculus

Rectangular/Cartesian Coordinates (x, y, z) $\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$$d\vec{l} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$$

$$d\vec{s}_x = dy dz \hat{a}_x \quad d\vec{s}_y = dx dz \hat{a}_y \quad d\vec{s}_z = dx dy \hat{a}_z$$

$$dv = dx dy dz$$

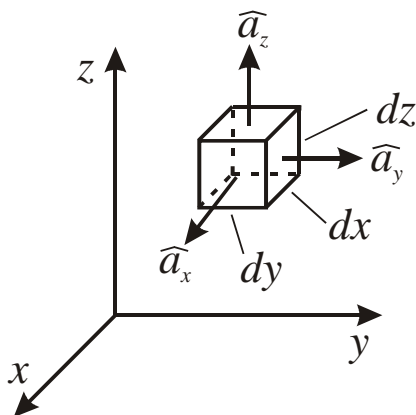
$$\vec{r} = x \hat{a}_x + y \hat{a}_y + z \hat{a}_z$$

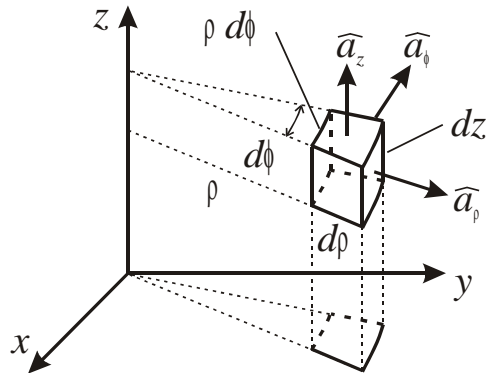
$$\vec{\nabla} \Phi = \hat{a}_x \frac{\partial \Phi}{\partial x} + \hat{a}_y \frac{\partial \Phi}{\partial y} + \hat{a}_z \frac{\partial \Phi}{\partial z}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla} \times \vec{A} = \hat{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2}$$





Cylindrical Coordinates (ρ, ϕ, z) $\bar{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$

$$d\bar{l} = d\rho \hat{a}_\rho + \rho d\phi \hat{a}_\phi + dz \hat{a}_z$$

$$d\bar{s}_\rho = \rho d\phi dz \hat{a}_\rho \quad d\bar{s}_\phi = d\rho dz \hat{a}_\phi \quad d\bar{s}_z = \rho d\rho d\phi \hat{a}_z$$

$$dv = \rho d\rho d\phi dz$$

$$\bar{r} = \rho \hat{a}_\rho + z \hat{a}_z \quad (\text{Note: } \hat{a}_\rho \text{ has } \phi \text{ dependence})$$

$$\bar{\nabla}\Phi = \hat{a}_\rho \frac{\partial\Phi}{\partial\rho} + \hat{a}_\phi \frac{1}{\rho} \frac{\partial\Phi}{\partial\phi} + \hat{a}_z \frac{\partial\Phi}{\partial z}$$

$$\bar{\nabla} \cdot \bar{A} = \frac{1}{\rho} \frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho} \frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\bar{\nabla} \times \bar{A} = \hat{a}_\rho \left(\frac{1}{\rho} \frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{a}_\phi \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho} \right) + \hat{a}_z \left(\frac{1}{\rho} \frac{\partial(\rho A_\phi)}{\partial\rho} - \frac{1}{\rho} \frac{\partial A_\rho}{\partial\phi} \right)$$

Spherical Coordinates (r, θ, ϕ) $\bar{A} = A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi$

$$d\bar{l} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin\theta d\phi \hat{a}_\phi$$

$$d\bar{s}_r = r^2 \sin\theta d\theta d\phi \hat{a}_r, \quad d\bar{s}_\theta = r \sin\theta dr d\phi \hat{a}_\theta$$

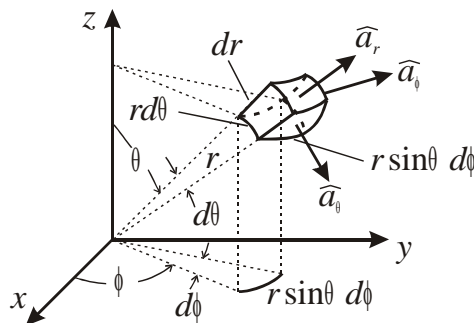
$$d\bar{s}_\phi = r dr d\theta \hat{a}_\phi \quad dv = r^2 \sin\theta dr d\theta d\phi$$

$$\bar{r} = r \hat{a}_r \quad (\text{Note: } \hat{a}_r \text{ has both } \theta \text{ \& } \phi \text{ dependence})$$

$$\bar{\nabla}\Phi = \hat{a}_r \frac{\partial\Phi}{\partial r} + \hat{a}_\theta \frac{1}{r} \frac{\partial\Phi}{\partial\theta} + \hat{a}_\phi \frac{1}{r \sin\theta} \frac{\partial\Phi}{\partial\phi}$$

$$\bar{\nabla} \cdot \bar{A} = \frac{1}{r^2} \frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial\phi}$$

$$\bar{\nabla} \times \bar{A} = \frac{\hat{a}_r}{r \sin\theta} \left[\frac{\partial(\sin\theta A_\phi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi} \right] + \hat{a}_\theta \left[\frac{1}{r \sin\theta} \frac{\partial A_r}{\partial\phi} - \frac{1}{r} \frac{\partial(r A_\phi)}{\partial r} \right] + \hat{a}_\phi \frac{1}{r} \left[\frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta} \right]$$



Gradient $\bar{\nabla}\Phi$ - vector representing magnitude & direction of maximum spatial rate of change of a scalar field

Divergence $\bar{\nabla} \cdot \bar{A}$ - net outward flux per unit volume of a vector field (scalar)

Flux of \bar{A} through surface s is $\int_s \bar{A} \cdot d\bar{s}$

Divergence Theorem- $\oint_s \bar{A} \cdot d\bar{s} = \int_v \bar{\nabla} \cdot \bar{A} dv$

Curl $\bar{\nabla} \times \bar{A}$ - circulation of a vector field per unit area w/ area oriented for maximize (vector whose direction is normal to surface by RHR)

Circulation of \bar{A} around contour c is $\oint_c \bar{A} \cdot d\bar{l}$

Stoke's Theorem- $\oint_c \bar{A} \cdot d\bar{l} = \int_s (\bar{\nabla} \times \bar{A}) \cdot d\bar{s}$

Vector field classification- $\bar{\nabla} \cdot \bar{A} = 0 \Rightarrow$ divergenceless/solenoidal; $\bar{\nabla} \times \bar{A} = 0 \Rightarrow$ irrotational & conservative

since $\oint_c \bar{A} \cdot d\bar{l} = \int_s (\bar{\nabla} \times \bar{A}) \cdot d\bar{s} = 0$

Cartesian Coordinates $(x, y, z) \Leftrightarrow$ **Cylindrical Coordinates** (ρ, ϕ, z)

Point/variable conversions :

$$\rho = \sqrt{x^2 + y^2} \quad x = \rho \cos \phi \quad \cos \phi = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\phi = \tan^{-1} \left(\frac{y}{x} \right) \quad y = \rho \sin \phi \quad \sin \phi = \frac{y}{\sqrt{x^2 + y^2}}$$

$$z = z \quad z = z$$

Vector conversions: $\vec{A} = \hat{a}_\rho A_\rho + \hat{a}_\phi A_\phi + A_z \hat{a}_z = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$$\hat{a}_x = \cos \phi \hat{a}_\rho - \sin \phi \hat{a}_\phi \quad \hat{a}_\rho = \cos \phi \hat{a}_x + \sin \phi \hat{a}_y$$

$$\hat{a}_y = \sin \phi \hat{a}_\rho + \cos \phi \hat{a}_\phi \quad \hat{a}_\phi = -\sin \phi \hat{a}_x + \cos \phi \hat{a}_y \quad \hat{a}_z = \hat{a}_z$$

$$A_\rho = A_x \cos \phi + A_y \sin \phi \quad A_x = A_\rho \cos \phi - A_\phi \sin \phi$$

$$A_\phi = -A_x \sin \phi + A_y \cos \phi \quad A_y = A_\rho \sin \phi + A_\phi \cos \phi \quad A_z = A_z$$

Dot Products :

$$\hat{a}_x \cdot \hat{a}_\rho = \cos \phi \quad \hat{a}_y \cdot \hat{a}_\rho = \sin \phi \quad \hat{a}_z \cdot \hat{a}_\rho = 0$$

$$\hat{a}_x \cdot \hat{a}_\phi = -\sin \phi \quad \hat{a}_y \cdot \hat{a}_\phi = \cos \phi \quad \hat{a}_z \cdot \hat{a}_\phi = 0$$

$$\hat{a}_x \cdot \hat{a}_z = 0 \quad \hat{a}_y \cdot \hat{a}_z = 0 \quad \hat{a}_z \cdot \hat{a}_z = 1$$

Cartesian Coordinates $(x, y, z) \Leftrightarrow$ **Spherical Coordinates** (r, θ, ϕ)

Point/variable conversions:

$$r = \sqrt{x^2 + y^2 + z^2} \quad x = r \sin \theta \cos \phi \quad \cos \phi = \frac{x}{\sqrt{x^2 + y^2}} \quad \cos \theta = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\phi = \tan^{-1} \frac{y}{x} \quad y = r \sin \theta \sin \phi$$

$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} \quad z = r \cos \theta \quad \sin \phi = \frac{y}{\sqrt{x^2 + y^2}} \quad \sin \theta = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}$$

Vector conversions: $\vec{A} = \hat{a}_r A_r + A_\theta \hat{a}_\theta + \hat{a}_\phi A_\phi = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$$\hat{a}_r = \sin \theta \cos \phi \hat{a}_x + \sin \theta \sin \phi \hat{a}_y + \cos \theta \hat{a}_z \quad \hat{a}_x = \sin \theta \cos \phi \hat{a}_r + \cos \theta \cos \phi \hat{a}_\theta - \sin \phi \hat{a}_\phi$$

$$\hat{a}_\theta = \cos \theta \cos \phi \hat{a}_x + \cos \theta \sin \phi \hat{a}_y - \sin \theta \hat{a}_z \quad \hat{a}_y = \sin \theta \sin \phi \hat{a}_r + \cos \theta \sin \phi \hat{a}_\theta + \cos \phi \hat{a}_\phi$$

$$\hat{a}_\phi = -\sin \phi \hat{a}_x + \cos \phi \hat{a}_y \quad \hat{a}_z = \cos \theta \hat{a}_r - \sin \theta \hat{a}_\theta$$

$$A_r = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta \quad A_x = A_r \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi$$

$$A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta \quad A_y = A_r \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi + A_\phi \cos \phi$$

$$A_\phi = -A_x \sin \phi + A_y \cos \phi \quad A_z = A_r \cos \theta - A_\theta \sin \theta$$

Dot Products:

$$\hat{a}_r \cdot \hat{a}_x = \sin \theta \cos \phi \quad \hat{a}_r \cdot \hat{a}_y = \sin \theta \sin \phi \quad \hat{a}_r \cdot \hat{a}_z = \cos \theta$$

$$\hat{a}_\theta \cdot \hat{a}_x = \cos \theta \cos \phi \quad \hat{a}_\theta \cdot \hat{a}_y = \cos \theta \sin \phi \quad \hat{a}_\theta \cdot \hat{a}_z = -\sin \theta$$

$$\hat{a}_\phi \cdot \hat{a}_x = -\sin \phi \quad \hat{a}_\phi \cdot \hat{a}_y = \cos \phi \quad \hat{a}_\phi \cdot \hat{a}_z = 0$$

Chapter 4 Electrostatic Fields

permittivity of free space: $\epsilon_0 = 8.8541878 \times 10^{-12}$ F/m

Coulomb's Law:

point charges $\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \hat{a}_{R12} = \frac{Q_1 Q_2 (\vec{r}_2 - \vec{r}_1)}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|^3}$ (N) force on Q_2 due to Q_1 (opposites attract, like repel)

Electric Field $\vec{E} = \vec{F} / q = -\nabla V$ is conservative (i.e., $\nabla \times \vec{E} = 0$ and $\oint_c \vec{E} \cdot d\vec{l} = 0$).

point charges: $\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{a}_R = \frac{Q(\vec{r} - \vec{r}')}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|^3}$ (V/m) & $\vec{E} = \sum_{k=1}^N \frac{Q_k(\vec{r} - \vec{r}_k)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_k|^3}$ (V/m)

line charge density: $\vec{E} = \int_{c'} \frac{\rho_l(\vec{r} - \vec{r}') d\vec{l}'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|^3}$ (V/m) where ρ_l can be a function of position (i.e. \vec{r}').

surface charge density: $\vec{E} = \int_{s'} \frac{\rho_s(\vec{r} - \vec{r}') ds'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|^3}$ (V/m) where ρ_s can be a function of position (i.e. \vec{r}').

volume charge density: $\vec{E} = \int_{v'} \frac{\rho_v(\vec{r} - \vec{r}') dv'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|^3}$ (V/m) where ρ_v can be a function of position (i.e. \vec{r}').

Electric Flux Density: $\vec{D} = \epsilon \vec{E} = \epsilon_r \epsilon_0 \vec{E}$ (C/m²) and Electric Flux: $\psi = \int \vec{D} \cdot d\vec{s}$ (C)

point charges: $\vec{D} = \frac{Q}{4\pi R^2} \hat{a}_R = \frac{Q(\vec{r} - \vec{r}')}{4\pi |\vec{r} - \vec{r}'|^3}$ (C/m²) and $\vec{D} = \sum_{k=1}^N \frac{Q_k(\vec{r} - \vec{r}_k)}{4\pi |\vec{r} - \vec{r}_k|^3}$ (C/m²)

line charge density: $\vec{D} = \int_{c'} \frac{\rho_l(\vec{r} - \vec{r}') d\vec{l}'}{4\pi |\vec{r} - \vec{r}'|^3}$ (C/m²) where ρ_l can be a function of position (i.e. \vec{r}').

surface charge density: $\vec{D} = \int_{s'} \frac{\rho_s(\vec{r} - \vec{r}') ds'}{4\pi |\vec{r} - \vec{r}'|^3}$ (C/m²) where ρ_s can be a function of position (i.e. \vec{r}').

volume charge density: $\vec{D} = \int_{v'} \frac{\rho_v(\vec{r} - \vec{r}') dv'}{4\pi |\vec{r} - \vec{r}'|^3}$ (C/m²) where ρ_v can be a function of position (i.e. \vec{r}').

Gauss' Law: Amount of electric flux through a closed surface is equal to amount of charge contained within surface. $\psi = \oint_s \vec{D} \cdot d\vec{s} = Q_{enc}$ (C). In differential/point form $\nabla \cdot \vec{D} = \rho_v$

Electrostatic potential V is work per unit charge to move from point A to B : $V_{AB} = V_B - V_A = \frac{W_{AB}}{q} = -\int_A^B \vec{E} \cdot d\vec{l}$

To get electric field from the electrostatic potential: $\vec{E} = -\nabla V$

point charges: $V = \frac{Q}{4\pi\epsilon_0 R} = \frac{Q}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|}$ (V) and $V = \sum_{k=1}^N \frac{Q_k}{4\pi\epsilon_0 |\vec{r} - \vec{r}_k|}$ (V) (wrt to infinity)

line charge density: $V = \int_{c'} \frac{\rho_l d\vec{l}'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|}$ (V) where ρ_l can be a function of position (i.e. \vec{r}').

surface charge density: $V = \int_{s'} \frac{\rho_s ds'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|}$ (V) where ρ_s can be a function of position (i.e. \vec{r}').

volume charge density: $V = \int_{v'} \frac{\rho_v dv'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|}$ (V) where ρ_v can be a function of position (i.e. \vec{r}').

Energy stored in an electric field: $W_E = \frac{1}{2} \sum_{k=1}^N Q_k V_k = \frac{1}{2} \int \bar{D} \cdot \bar{E} dv = \frac{1}{2} \int \epsilon_r \epsilon_0 |\bar{E}|^2 dv = \frac{1}{2} CV^2$ (J)

Energy density of an electric field: $w_E = \frac{1}{2} \bar{D} \cdot \bar{E} = \frac{1}{2} \epsilon_r \epsilon_0 |\bar{E}|^2$ (J/m³)

Chapter 5 Electric Fields in Material Space

Conduction current density: $\bar{J} = \sigma \bar{E}$ (A/m²) and electrical current $I = \int_s \bar{J} \cdot d\bar{s} = \frac{dQ}{dt}$ (A)

Resistance: $R = \frac{V}{I} = \frac{l}{\sigma S}$ (Ω) where l is length, σ is conductivity (S/m), and S is surface area

Perfect electrical conductor (PEC)- $\bar{E}_{\text{inside}} = 0$, $\rho_{v, \text{inside}} = 0$, and $V_{ab} = 0$ for any two points a and b inside.

Power dissipation: $P = \int \bar{E} \cdot \bar{J} dv = \int \sigma |\bar{E}|^2 dv$ (W) and power density $w_p = \bar{E} \cdot \bar{J} = \sigma |\bar{E}|^2$ (W/m³)

Polarization vector: $\bar{P} = \chi_e \epsilon_0 \bar{E}$ relates electric field & electric flux density in materials per $\bar{D} = \epsilon_0 \bar{E} + \bar{P}$.

Permittivity: $\epsilon = (1 + \chi_e) \epsilon_0 = \epsilon_r \epsilon_0$ where χ_e is electric susceptibility and ϵ_r is the relative permittivity.

Bound surface charge density $\rho_{ps} = \bar{P} \cdot \hat{a}_n$ and bound volume charge density $\rho_{pv} = -\bar{\nabla} \cdot \bar{P}$ can be used to represent polarized materials.

Equation of Continuity $I_{\text{out}} = \oint_s \bar{J} \cdot d\bar{s} = \frac{-dQ_{\text{inside}}}{dt}$ or $\bar{\nabla} \cdot \bar{J} = \frac{-\partial \rho_v}{\partial t}$

Charge density in conductive materials $\rho_v = \rho_v(0) e^{-t/T_r}$ where $T_r = \epsilon / \sigma$ is the relaxation time.

Capacitance: Relates stored charge to potential difference $C = \frac{Q}{V}$ (e.g., parallel-plate $C = \frac{\epsilon S}{d}$)

Electrostatic Boundary conditions between dielectric regions

Tangential- $E_{1t} = \frac{D_{1t}}{\epsilon_1} = E_{2t} = \frac{D_{2t}}{\epsilon_2}$, $\bar{E}_{1t} = \bar{E}_{2t}$, or $\hat{a}_{n12} \times (\bar{E}_2 - \bar{E}_1) = 0$ and

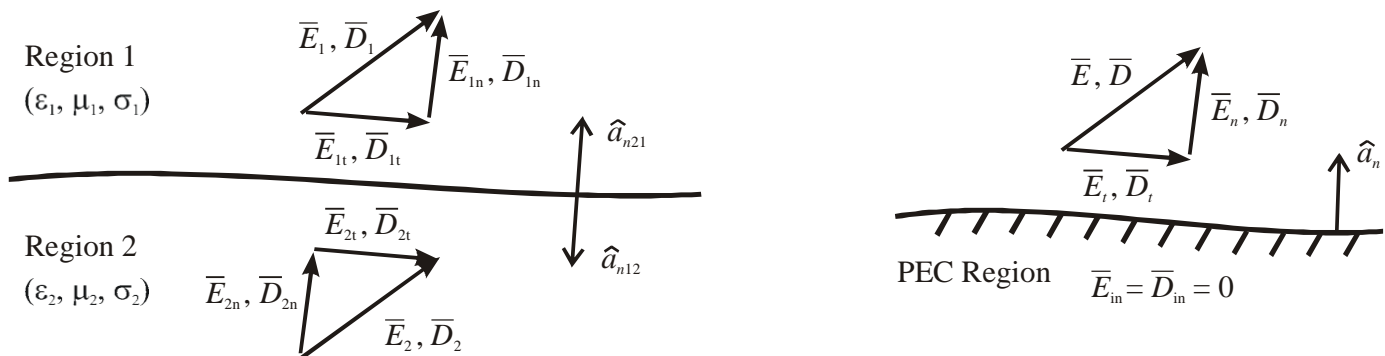
Normal- $D_{1n} - D_{2n} = \epsilon_1 E_{1n} - \epsilon_2 E_{2n} = \rho_s$ or $\hat{a}_{n12} \cdot (\bar{D}_2 - \bar{D}_1) = \hat{a}_{n21} \cdot (\bar{D}_1 - \bar{D}_2) = \rho_s$.

If $\rho_s = 0$, $D_{1n} = D_{2n}$ or $\hat{a}_{n12} \cdot (\bar{D}_2 - \bar{D}_1) = 0$. Surface normal \hat{a}_{n12} points from region 1 into region 2, and \mathcal{D}_{1n} points away from boundary while \mathcal{D}_{2n} points toward the boundary

Electrostatic Boundary conditions at dielectric-PEC interface

Tangential- $E_t = 0$, $\bar{E}_t = 0$, or $\hat{a}_n \times \bar{E} = 0$ and Normal- $D_n = \rho_s$ or $\hat{a}_n \cdot \bar{D} = \rho_s$.

Surface normal \hat{a}_n points from PEC region into dielectric region, i.e., \mathcal{D}_n points away PEC.



Chapter 6 Electrostatic Boundary Value Problems

Based on solving problems where the potential at the boundaries and materials inside the boundaries are known using Poisson's equation $\nabla^2 V = -\rho_v/\epsilon$ or Laplace's equation $\nabla^2 V = 0$.

$$\text{Capacitance- } C = \frac{Q}{V} = \frac{\oint_s \bar{D} \cdot d\bar{s}}{-\int_{\text{low}}^{\text{high}} \bar{E} \cdot d\bar{l}} = \frac{\epsilon \oint_s \bar{E} \cdot d\bar{s}}{-\int_{\text{low}}^{\text{high}} \bar{E} \cdot d\bar{l}} \quad \text{Resistance- } R = \frac{V}{I} = \frac{-\int_{\text{low}}^{\text{high}} \bar{E} \cdot d\bar{l}}{\oint_s \bar{J} \cdot d\bar{s}} = \frac{-\int_{\text{low}}^{\text{high}} \bar{E} \cdot d\bar{l}}{\sigma \oint_s \bar{E} \cdot d\bar{s}}$$

Resistance and capacitance are related by $RC = \epsilon/\sigma$.

Method of Images- replace conducting planes with image charges so that boundary conditions still satisfied.

Chapter 7 Magnetostatic Fields

permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ H/m

Biot-Savart Law:

$$\text{Filamentary current } \bar{H} = \oint_{c'} \frac{I d\bar{l}' \times \hat{a}_R}{4\pi R^2} = \oint_{c'} \frac{I d\bar{l}' \times (\bar{r} - \bar{r}')}{4\pi |\bar{r} - \bar{r}'|^3} \quad (\text{A/m})$$

$$\text{Surface current density } \bar{H} = \oint_{s'} \frac{\bar{J}_s ds' \times \hat{a}_R}{4\pi R^2} = \oint_{c'} \int_{w'} \frac{J_s dw' d\bar{l}' \times (\bar{r} - \bar{r}')}{4\pi |\bar{r} - \bar{r}'|^3} \quad (\text{A/m})$$

$$\text{Volume current density } \bar{H} = \oint_{v'} \frac{\bar{J} dv' \times \hat{a}_R}{4\pi R^2} = \oint_{c'} \int_{s'} \frac{J ds' d\bar{l}' \times (\bar{r} - \bar{r}')}{4\pi |\bar{r} - \bar{r}'|^3} \quad (\text{A/m})$$

$$\text{For a long straight line current: } \bar{H} = \frac{I}{2\pi\rho} \hat{a}_\phi \quad (\text{A/m})$$

Magnetic Flux Density: $\bar{B} = \mu_r \mu_0 \bar{H}$ (Wb/m² or T) and Magnetic Flux: $\psi_m = \int \bar{B} \cdot d\bar{s}$ (Wb)

Ampere's Law: Line integral of the magnetic field around a closed path is equal to the amount of current passing through the surface enclosed. $\oint_c \bar{H} \cdot d\bar{l} = I_{\text{enclosed}}$ (A). Diff/point form $\nabla \times \bar{H} = \bar{J}$.

Vector Magnetic Potential \bar{A} : related to magnetic flux density by $\bar{B} = \nabla \times \bar{A}$ and to magnetic flux by

$$\psi_m = \oint_c \bar{A} \cdot d\bar{l} \quad (\text{Wb})$$

$$\text{Filamentary current } \bar{A} = \oint_{c'} \frac{\mu I d\bar{l}'}{4\pi R} = \oint_{c'} \frac{\mu I d\bar{l}'}{4\pi |\bar{r} - \bar{r}'|} \quad (\text{Wb/m})$$

$$\text{Surface current density } \bar{A} = \oint_{s'} \frac{\mu \bar{J}_s ds'}{4\pi R} = \oint_{c'} \int_{w'} \frac{\mu J_s dw' d\bar{l}'}{4\pi |\bar{r} - \bar{r}'|} \quad (\text{Wb/m})$$

$$\text{Volume current density } \bar{A} = \oint_{v'} \frac{\mu \bar{J} dv'}{4\pi R} = \oint_{c'} \int_{s'} \frac{\mu J ds' d\bar{l}'}{4\pi |\bar{r} - \bar{r}'|} \quad (\text{Wb/m})$$

Maxwell's Equations for Static Fields

	<u>Integral Form</u>	<u>Differential Form</u>
Faraday's Law	$\oint_c \vec{E} \cdot d\vec{l} = 0$	$\vec{\nabla} \times \vec{E} = 0$
Ampere's Law	$\oint_c \vec{H} \cdot d\vec{l} = \int_s \vec{J} \cdot d\vec{s}$	$\vec{\nabla} \times \vec{H} = \vec{J}$
Gauss' Law	$\oint_s \vec{D} \cdot d\vec{s} = \int_v \rho_v dV$	$\vec{\nabla} \cdot \vec{D} = \rho_v$
	$\oint_s \vec{B} \cdot d\vec{s} = 0$	$\vec{\nabla} \cdot \vec{B} = 0$

Chapter 8 Magnetic Forces, Materials, and Devices

Force of a magnetic field on a point charge: $\vec{F}_m = Q\vec{u} \times \vec{B} = m\vec{a}$

Lorentz force equation: $\vec{F} = \vec{F}_e + \vec{F}_m = Q(\vec{E} + \vec{u} \times \vec{B}) = m\vec{a}$

Force of a magnetic field on current elements:

filamentary current: $\vec{F} = \oint_{c'} I d\vec{l}' \times \vec{B}$

surface current density- $\vec{F} = \oint_{c'} \int_{w'} \vec{J}_s d\vec{l}' dw' \times \vec{B}$

volume current density- $\vec{F} = \oint_{c'} \int_{s'} \vec{J} d\vec{l}' ds' \times \vec{B}$

Force between two filamentary current elements (force exerted by element 2 on 1, $\vec{F}_{21} = -\vec{F}_{12}$):

filamentary currents- $\vec{F}_{21} = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{c_1} \oint_{c_2} \frac{d\vec{l}_1 \times [d\vec{l}_2 \times (\vec{r}_1 - \vec{r}_2)]}{|\vec{r}_1 - \vec{r}_2|^3}$

Torque- $\vec{T} = \vec{r} \times \vec{F} = \vec{m} \times \vec{B}$ (N·m) where \vec{r} is the moment arm, \vec{F} is force, and $\vec{m} = IS\hat{a}_n$ is the magnetic dipole moment.

Magnetization vector $\vec{M} = \chi_m \vec{H}$ relates magnetic field and magnetic flux density in materials per $\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (1 + \chi_m) \vec{H} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$.

Magnetostatic Boundary conditions between magnetic regions

Tangential- $\hat{a}_{n12} \times (\vec{H}_2 - \vec{H}_1) = \hat{a}_{n21} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$. If $\vec{J}_s = 0$, then $\vec{H}_{1t} = \vec{H}_{2t}$

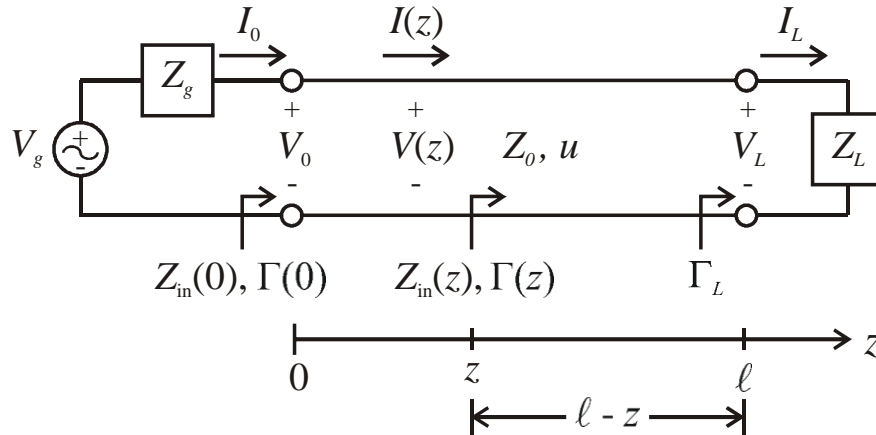
Normal- $\vec{B}_{1n} = \vec{B}_{2n}$ or $\hat{a}_{n12} \cdot (\vec{B}_1 - \vec{B}_2) = \hat{a}_{n21} \cdot (\vec{B}_2 - \vec{B}_1) = 0$.

Surface normal \hat{a}_{n12} points from region 1 into 2, and B_{1n} points away from boundary while B_{2n} points toward from boundary

Energy stored in a magnetic field: $W_M = \frac{1}{2} \int \vec{B} \cdot \vec{H} dv = \frac{1}{2} \int \mu_r \mu_0 |\vec{H}|^2 dv = \frac{1}{2} LI^2$ (J)

Energy density of a magnetic field: $w_M = \frac{1}{2} \vec{B} \cdot \vec{H} = \frac{1}{2} \mu_r \mu_0 |\vec{H}|^2$ (J/m³)

Chapter 11 Lossless Transmission Lines



Phase velocity $u = f\lambda = \frac{\omega}{\beta} = \frac{1}{\sqrt{lc}}$ where l and c are the per-unit-length inductance and capacitance

Phase constant $\beta = \frac{\omega}{u} = \omega\sqrt{lc} = \frac{2\pi}{\lambda}$ (rad/m), Electrical length βl (rad) or $(\beta l)\frac{180^\circ}{\pi}$ (deg) or $(\beta l)\frac{\lambda}{2\pi}$

Wavelength $\lambda = \frac{u}{f} = \frac{2\pi}{\beta}$ (m), Characteristic impedance $Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \sqrt{\frac{l}{c}}$ (Ω)

Phasor voltage & current $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$ (V) where V_0^+, V_0^-, I_0^+ , and I_0^- are the forward and backward traveling voltages and currents.
 $I(z) = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}$ (A)

Reflection Coefficient: $\Gamma(z) = \frac{V_{ref}(z)}{V_{inc}(z)} = -\frac{I_{ref}(z)}{I_{inc}(z)} = \frac{Z_{in}(z) - Z_0}{Z_{in}(z) + Z_0} = \Gamma_L e^{-j2\beta(\ell-z)}$, of particular interest are the load

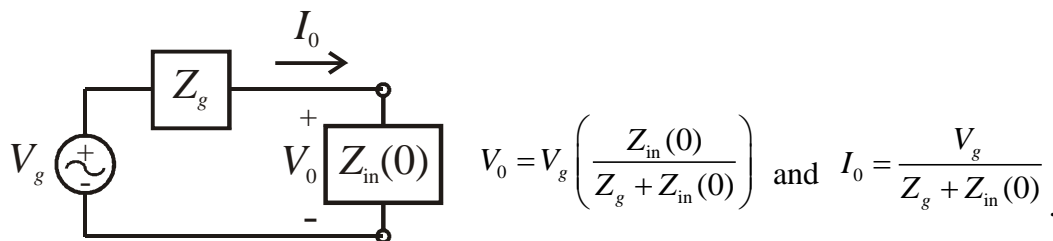
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \text{ and input } \Gamma(0) = \frac{Z_{in}(0) - Z_0}{Z_{in}(0) + Z_0} = \Gamma_L e^{-j2\beta\ell} \text{ reflection coefficients.}$$

$$\text{Input impedance } Z_{in}(z) = \frac{V(z)}{I(z)} = Z_0 \left[\frac{Z_L + jZ_0 \tan[\beta(\ell-z)]}{Z_0 + jZ_L \tan[\beta(\ell-z)]} \right] = Z_0 \left[\frac{1 + \Gamma(z)}{1 - \Gamma(z)} \right]$$

$$\text{Standing wave ratio } SWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Input Power $P_{ave}(z) = 0.5 \text{Re}\{V(z)I^*(z)\}$ (W) (constant on a lossless transmission line)

The input voltage $V(0) = V_0$ and current $I(0) = I_0$ can be found using the equivalent circuit-



Note: $V_0 = V_0^+ + V_0^-$, $V_0^+ = 0.5(V_0 + I_0 Z_0) = \frac{V_0}{1 + \Gamma(0)}$, and $V_0^- = 0.5(V_0 - I_0 Z_0) = V_0 - V_0^+$.