$\underline{\text{Cartesian Coordinates}} \Leftrightarrow \underline{\text{Spherical Coordinates}}$

$$(x, y, z) \Leftrightarrow (r, \theta, \phi)$$

Point/variable conversions:

$$r = \sqrt{x^2 + y^2 + z^2} \qquad x = r\sin\theta\cos\phi \qquad \cos\phi = \frac{x}{\sqrt{x^2+y^2}} \qquad \cos\theta = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\phi = \tan^{-1}\frac{y}{x} \qquad y = r\sin\theta\sin\phi$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} \qquad z = r\cos\theta \qquad \sin\phi = \frac{y}{\sqrt{x^2+y^2}} \qquad \sin\theta = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$$

Vector conversions:

$$\overline{A} = \hat{a}_r A_r + A_\theta \hat{a}_\theta + \hat{a}_\phi A_\phi = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

$$\hat{a}_r = \sin\theta\cos\phi\,\hat{a}_x + \sin\theta\sin\phi\,\hat{a}_y + \cos\theta\,\hat{a}_z$$

$$\hat{a}_\theta = \cos\theta\cos\phi\,\hat{a}_x + \cos\theta\sin\phi\,\hat{a}_y - \sin\theta\,\hat{a}_z$$

$$\hat{a}_\phi = -\sin\phi\,\hat{a}_x + \cos\phi\,\hat{a}_y$$

$$\hat{a}_x = \sin\theta\cos\phi\,\hat{a}_r + \cos\theta\cos\phi\,\hat{a}_\theta - \sin\phi\,\hat{a}_\phi$$

$$\hat{a}_y = \sin\theta\sin\phi\,\hat{a}_r + \cos\theta\sin\phi\,\hat{a}_\theta + \cos\phi\,\hat{a}_\phi$$

$$\hat{a}_z = \cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta$$

$$A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$$

$$A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$$

$$A_\phi = -A_x\sin\phi + A_y\cos\phi$$

$$A_x = A_r\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$$

$$A_y = A_r\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$$

$$A_z = A_r\cos\theta - A_\theta\sin\theta$$

Dot Products:

$$\hat{a}_r \bullet \hat{a}_x = \sin\theta\cos\phi \qquad \hat{a}_r \bullet \hat{a}_y = \sin\theta\sin\phi \qquad \hat{a}_r \bullet \hat{a}_z = \cos\theta$$

$$\hat{a}_\theta \bullet \hat{a}_x = \cos\theta\cos\phi \qquad \hat{a}_\theta \bullet \hat{a}_y = \cos\theta\sin\phi \qquad \hat{a}_\theta \bullet \hat{a}_z = -\sin\theta$$

$$\hat{a}_\phi \bullet \hat{a}_x = -\sin\phi \qquad \hat{a}_\phi \bullet \hat{a}_y = \cos\phi \qquad \hat{a}_\phi \bullet \hat{a}_z = 0$$