EE 381 Electric and Magnetic Fields Quiz #10 (Fall 2026)

Name ___KEY A___

Instructions: **Open book & notes**. Show all work for full credit, and use notation & units as given in class. Place answers in indicated spaces.

In moist soil ($\varepsilon = 12\varepsilon_0$, μ_0, $\sigma = 0.5$ S/m), the potential is given by $V = 60x^2 - 80y^2 + 20z^2 - 200$ (V). Does Laplace's or Poisson's equation apply to this soil? Justify your answer. Determine a general expression for the vector electric field in the soil. Then, find the vector electric flux density and current density as well as the free volume charge density at the point $S(x = 1.8\text{ m}, y = 1.6\text{ m}, z = 0)$.

Compute the scalar Laplacian of V in Cartesian coordinates

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\partial(120x)}{\partial x} + \frac{\partial(-160y)}{\partial y} + \frac{\partial(40z)}{\partial z}.$$
$$= 120 - 160 + 40 = 0$$

Per (6.5), since $\nabla^2 V = 0$, **Laplace's equation applies**.

Per (4.76), $\bar{E} = -\bar{\nabla} V = -\hat{a}_x \frac{\partial V}{\partial x} - \hat{a}_y \frac{\partial V}{\partial y} - \hat{a}_z \frac{\partial V}{\partial z} = \underline{-\hat{a}_x 120x + \hat{a}_y 160y - \hat{a}_z 40z \text{ (V/m)}}$.

At point $S(x = 1.8\text{ m}, y = 1.6\text{ m}, z = 0)$

Per (5.34), $\bar{D}_S = \varepsilon \bar{E}_S = 12(8.8541878 \times 10^{-12})[-\hat{a}_x 120(1.8) + \hat{a}_y 160(1.6) - \hat{a}_z 40(0)]$

$$\Rightarrow \underline{\bar{D}_S = -\hat{a}_x 2.2950055 \times 10^{-8} + \hat{a}_y 2.7200065 \times 10^{-8} \text{ (C/m}^2\text{)}}.$$

Per (5.11), $\bar{J}_S = \sigma \bar{E}_S = 0.5[-\hat{a}_x 120(1.8) + \hat{a}_y 160(1.6) - \hat{a}_z 40(0)]$

$$\Rightarrow \underline{\bar{J}_S = -108\hat{a}_x + 128\hat{a}_y \text{ (A/m}^2\text{)}}.$$

Per (6.4), since $\nabla^2 V = -\rho_v / \varepsilon = 0 \quad \Rightarrow \quad \underline{\rho_{v,S} = 0}$.

Can get same result using Gauss' Law (6.1)

$$\rho_{v,S} = (\bar{\nabla} \bullet \bar{D})_S = (\bar{\nabla} \bullet \varepsilon \bar{E})_S = 12(8.8541878 \times 10^{-12})[-120 + 160 - 40] \Rightarrow \underline{\rho_{v,S} = 0}.$$

Laplace's equation or Poisson's equation? (Circle correct answer) Why? $\underline{\nabla^2 V = 0}$

$\bar{E} = \underline{-120x\hat{a}_x + 160y\hat{a}_y - 40z\hat{a}_z \text{ (V/m)}}$ $\quad$ $\bar{D}_S = \underline{-22.95\hat{a}_x + 27.20\hat{a}_y \text{ (nC/m}^2\text{)}}$

$\bar{J}_S = \underline{-108\hat{a}_x + 128\hat{a}_y \text{ (A/m}^2\text{)}}$ $\quad$ free vol. charge density @ $S = \underline{\rho_{v,S} = 0}$

EE 381 Electric and Magnetic Fields Quiz #10 (Fall 2026)

Name ___KEY B___

Instructions: **Open book & notes**. Show all work for full credit, and use notation & units as given in class. Place answers in indicated spaces.

In wet dirt ($\varepsilon = 16\varepsilon_0$, μ_0, $\sigma = 0.3$ S/m), the potential is given by $V = -60x^2 + 80y^2 - 20z^2 + 800$ (V). Does Laplace's or Poisson's equation apply to this dirt? Justify your answer. Determine a general expression for the vector electric field in the dirt. Then, find the vector electric flux density and current density as well as the free volume charge density at the point $D(x = 0, y = 1.4 \text{ m}, z = 1.8 \text{ m})$.

Compute the scalar Laplacian of V in Cartesian coordinates

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\partial(-120x)}{\partial x} + \frac{\partial(160y)}{\partial y} + \frac{\partial(-40z)}{\partial z} .$$
$$= -120 + 160 - 40 = 0$$

Per (6.5), since $\nabla^2 V = 0$, **Laplace's equation applies**.

Per (4.76), $\bar{E} = -\bar{\nabla} V = -\hat{a}_x \frac{\partial V}{\partial x} - \hat{a}_y \frac{\partial V}{\partial y} - \hat{a}_z \frac{\partial V}{\partial z} = \hat{a}_x 120x - \hat{a}_y 160y + \hat{a}_z 40z \text{ (V/m)}$.

At point $D(x = 0, y = 1.4 \text{ m}, z = 1.8 \text{ m})$

Per (5.34), $\bar{D}_D = \varepsilon \bar{E}_D = 16(8.8541878 \times 10^{-12})[\hat{a}_x 120(0) - \hat{a}_y 160(1.4) + \hat{a}_z 40(1.8)]$

$$\Rightarrow \bar{D}_S = -\hat{a}_y 3.173341 \times 10^{-8} + \hat{a}_z 1.020002 \times 10^{-8} \text{ (C/m}^2\text{)}.$$

Per (5.11), $\bar{J}_D = \sigma \bar{E}_D = 0.3[\hat{a}_x 120(0) - \hat{a}_y 160(1.4) + \hat{a}_z 40(1.8)]$

$$\Rightarrow \bar{J}_D = -67.2\hat{a}_y + 21.6\hat{a}_z \text{ (A/m}^2\text{)}.$$

Per (6.4), since $\nabla^2 V = -\rho_v / \varepsilon = 0 \quad \Rightarrow \quad \rho_{v,D} = 0$.

Can get same result using Gauss' Law (6.1)

$$\rho_{v,D} = (\bar{\nabla} \bullet \bar{D})_D = (\bar{\nabla} \bullet \varepsilon \bar{E})_S = 12(8.8541878 \times 10^{-12})[120 - 160 + 40] \Rightarrow \rho_{v,D} = 0.$$

Laplace's equation or Poisson's equation? (Circle correct answer) Why? $\nabla^2 V = 0$

$\bar{E} = 120x\hat{a}_x - 160y\hat{a}_y + 40z\hat{a}_z$ (V/m) $\quad \bar{D}_D = -31.733\hat{a}_y + 10.200\hat{a}_z$ (nC/m^2)

$\bar{J}_D = -67.2\hat{a}_y + 21.6\hat{a}_z$ (A/m^2) $\quad$ free vol. charge density @ $D = \rho_{v,D} = 0$