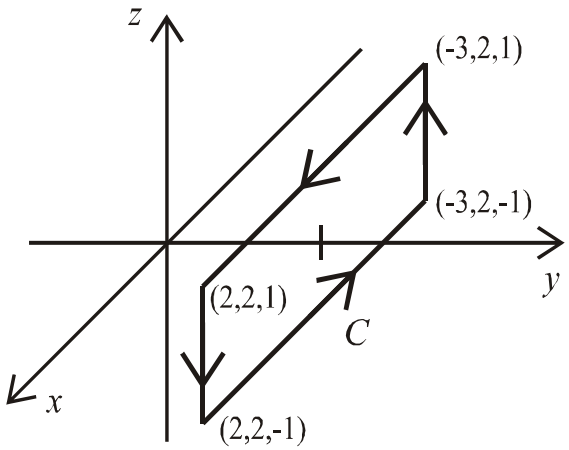
EE 381 Electric and Magnetic Fields Quiz #6 (Fall 2026)

Name Key Tapir

Instructions: **Closed** book & notes. Place answers in indicated spaces and show all work for credit.

Given vector field $\overline{T} = 4xz^2\hat{a}_x - 6zy^2\hat{a}_y - 8\hat{a}_z$ (Tapirs/m), calculate the divergence and curl of $\overline{T}$. Then, calculate the circulation of $\overline{T}$ about closed contour C shown below. All locations are in meters.

$$\overline{\nabla}\bullet\overline{T} = \frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} + \frac{\partial T_z}{\partial z}$$
$$= \frac{\partial(4xz^2)}{\partial x} + \frac{\partial(-6zy^2)}{\partial y} + \frac{\partial(-8)}{\partial z}$$
$$= 4z^2 - 12yz + 0 \ \ (\text{Tapirs/m}^2)$$
$$= 4z(z-3y) \ \ (\text{Tapirs/m}^2)$$



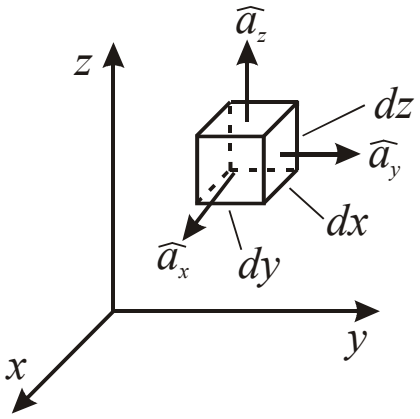
$$\overline{\nabla}\times\overline{T} = \hat{a}_x\left(\frac{\partial T_z}{\partial y} - \frac{\partial T_y}{\partial z}\right) + \hat{a}_y\left(\frac{\partial T_x}{\partial z} - \frac{\partial T_z}{\partial x}\right) + \hat{a}_z\left(\frac{\partial T_y}{\partial x} - \frac{\partial T_x}{\partial y}\right)$$
$$= \hat{a}_x\left(\frac{\partial(-8)}{\partial y} - \frac{\partial(-6zy^2)}{\partial z}\right) + \hat{a}_y\left(\frac{\partial(4xz^2)}{\partial z} - \frac{\partial(-8)}{\partial x}\right) + \hat{a}_z\left(\frac{\partial(-6zy^2)}{\partial x} - \frac{\partial(4xz^2)}{\partial y}\right)$$
$$= \hat{a}_x\left(0 + 6y^2\right) + \hat{a}_y\left(8xz - 0\right) + \hat{a}_z\left(0 - 0\right) = 6y^2\hat{a}_x + 8xz\hat{a}_y \ \ (\text{Tapirs/m}^2)$$

Use Stoke's Theorem as vector is well-behaved on contour -

$$\oint_C \overline{T}\bullet d\overline{l} = \iint_S (\overline{\nabla}\times\overline{T})\bullet d\overline{s} = \iint_S (6y^2\hat{a}_x + 8xz\hat{a}_y)\bullet d\overline{s}_y = \iint_S 8x\,z\,dxdz$$
$$= 8\int_{-3}^{2} x\,dx \int_{-1}^{1} z\,dz = 2\left(x^2\right)\Big|_{-3}^{2}\left(z^2\right)\Big|_{-1}^{1} = 2\left(2^2 - (-3)^2\right)\left(1^2 - (-1)^2\right)$$
$$= 0$$

div $\overline{T}$ = **$4z^2 - 12yz = 4z(z - 3y)$ (Tapirs/m^2)**

curl $\overline{T}$ = **$6y^2\,\hat{a}_x + 8xz\,\hat{a}_y$ (Tapirs/m^2)** Circulation of $\overline{T}$ about C = **0**



Rectangular Coordinates (x, y, z) $\quad \bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$$d\bar{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

$$d\bar{s}_x = dy\,dz\,\hat{a}_x \qquad d\bar{s}_y = dx\,dz\,\hat{a}_y \qquad d\bar{s}_z = dx\,dy\,\hat{a}_z$$

$$dv = dx\,dy\,dz$$

$$\bar{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$$

$$\bar{\nabla}\Phi = \hat{a}_x \frac{\partial \Phi}{\partial x} + \hat{a}_y \frac{\partial \Phi}{\partial y} + \hat{a}_z \frac{\partial \Phi}{\partial z}$$

$$\bar{\nabla} \bullet \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\bar{\nabla} \times \bar{A} = \hat{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2}$$

Divergence Theorem- $\oint_s \bar{A} \bullet d\bar{s} = \int_v \bar{\nabla} \bullet \bar{A}\, dv$ Stoke's Theorem- $\oint_c \bar{A} \bullet d\bar{l} = \int_s (\bar{\nabla} \times \bar{A}) \bullet d\bar{s}$

Flux of $\bar{A}$ through surface s is $\int_s \bar{A} \bullet d\bar{s}$

Circulation of $\bar{A}$ around contour c is $\oint_c \bar{A} \bullet d\bar{l}$

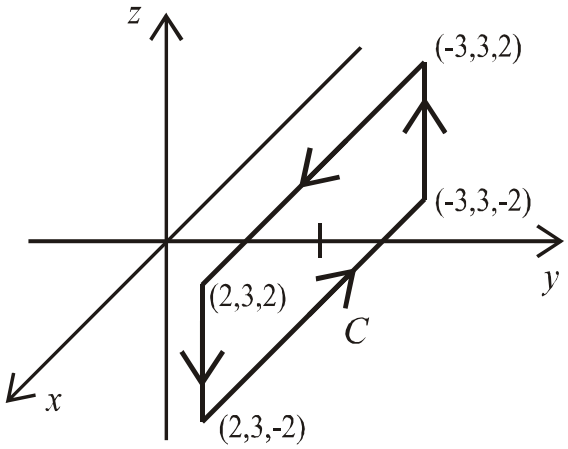
Vector field classification- $\bar{\nabla} \bullet \bar{A} = 0 \Rightarrow$ divergenceless/solenoidal; $\bar{\nabla} \times \bar{A} = 0 \Rightarrow$ irrotational & conservative since $\oint_c \bar{A} \bullet d\bar{l} = \int_s (\bar{\nabla} \times \bar{A}) \bullet d\bar{s} = 0$

EE 381 Electric and Magnetic Fields Quiz #6 (Fall 2026)

Name Key Macaw

Instructions: **Closed** book & notes. Place answers in indicated spaces and show all work for credit.

Given vector field $\bar{M} = -3xz^2\hat{a}_x + 6zy^2\hat{a}_y + 9\hat{a}_z$ (Macaws/m), calculate the divergence and curl of $\bar{M}$. Then, calculate the circulation of $\bar{M}$ about closed contour C shown below. All locations are in meters.



$$\bar{\nabla}\bullet\bar{M} = \frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} + \frac{\partial M_z}{\partial z}$$
$$= \frac{\partial(-3xz^2)}{\partial x} + \frac{\partial(6zy^2)}{\partial y} + \frac{\partial(9)}{\partial z}$$
$$= -3z^2 + 12yz + 0 \ \ (\text{Macaws/m}^2)$$
$$= -3z(z-4y) \ \ (\text{Macaws/m}^2)$$

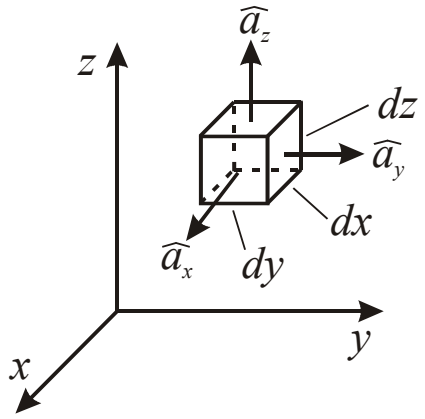
$$\bar{\nabla}\times\bar{M} = \hat{a}_x\left(\frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}\right) + \hat{a}_y\left(\frac{\partial M_x}{\partial z} - \frac{\partial M_z}{\partial x}\right) + \hat{a}_z\left(\frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y}\right)$$
$$= \hat{a}_x\left(\frac{\partial(9)}{\partial y} - \frac{\partial(6zy^2)}{\partial z}\right) + \hat{a}_y\left(\frac{\partial(-3xz^2)}{\partial z} - \frac{\partial(9)}{\partial x}\right) + \hat{a}_z\left(\frac{\partial(6zy^2)}{\partial x} - \frac{\partial(-3xz^2)}{\partial y}\right)$$
$$= \hat{a}_x\left(0 - 6y^2\right) + \hat{a}_y\left(-6xz - 0\right) + \hat{a}_z\left(0 - 0\right) = -6y^2\hat{a}_x - 6xz\hat{a}_y \ \ (\text{Macaws/m}^2)$$

Use Stoke's Theorem as vector is well-behaved on contour-

$$\oint_C \bar{M}\bullet d\bar{l} = \iint_S (\bar{\nabla}\times\bar{M})\bullet d\bar{s} = \iint_S (-6y^2\hat{a}_x - 6xz\hat{a}_y)\bullet d\bar{s}_y = \iint_S -6x\,z\,dxdz$$
$$= -6\int_{-3}^{2} x\,dx \int_{-2}^{2} z\,dz = \frac{-6}{2(2)}\left(x^2\right)\Big|_{-3}^{2}\left(z^2\right)\Big|_{-2}^{2} = -1.5\left(2^2-(-3)^2\right)\left(2^2-(-2)^2\right)$$
$$= 0$$

div $\bar{M}$ = **$-3z^2 + 12yz = -3z\,(z - 4y)$ (Macaws/m^2)**

curl $\bar{M}$ = **$-6y^2\hat{a}_x - 6xz\hat{a}_y$ (Macaws/m^2)** Circulation of $\bar{M}$ about C = **0**



Rectangular Coordinates (x, y, z) $\quad \bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$$d\bar{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

$$d\bar{s}_x = dy\,dz\,\hat{a}_x \qquad d\bar{s}_y = dx\,dz\,\hat{a}_y \qquad d\bar{s}_z = dx\,dy\,\hat{a}_z$$

$$dv = dx\,dy\,dz$$

$$\bar{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$$

$$\bar{\nabla}\Phi = \hat{a}_x \frac{\partial \Phi}{\partial x} + \hat{a}_y \frac{\partial \Phi}{\partial y} + \hat{a}_z \frac{\partial \Phi}{\partial z}$$

$$\bar{\nabla} \bullet \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\bar{\nabla} \times \bar{A} = \hat{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2}$$

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