

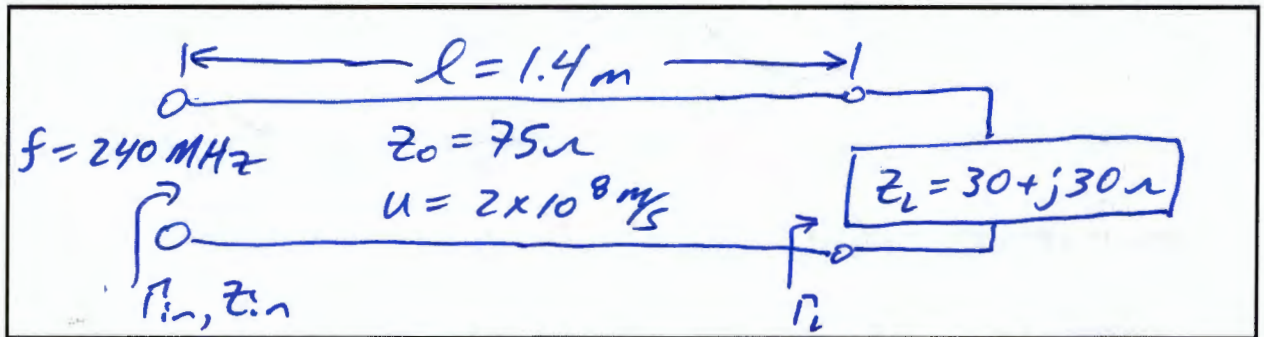
EE 381 Electric and Magnetic Fields Quiz #2 (Fall 2026)

Name

Key A

Instructions: **Closed** book & notes (see back). Place answers in indicated spaces and show all work for credit.

A lossless transmission line has a characteristic impedance of 75Ω and phase velocity of 2×10^8 m/s. If it is operated at 240 MHz, find the phase constant for this transmission line. If a 1.4 m long section of the transmission line is terminated with a $30 + j30 \Omega$ load, sketch a fully labeled drawing of this transmission line circuit in box. Then, find the load and input reflection coefficients and input impedance. [Note: Express currents, voltages, & reflection coefficients in polar format w/ angles in degrees. Express impedances in rectangular format.]



$$\beta = \frac{\omega}{u} = \frac{2\pi(240 \times 10^6)}{2 \times 10^8} = 7.5398 \text{ rad/m}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 + j30) - 75}{(30 + j30) + 75} = 0.4953 \angle 130.365^\circ$$

$$\Gamma_{in} = \Gamma_L e^{-j2\beta l} = (0.4953 \angle 130.365^\circ) e^{-j2(7.54)(1.4)} \\ = 0.4953 \angle 0.765^\circ$$

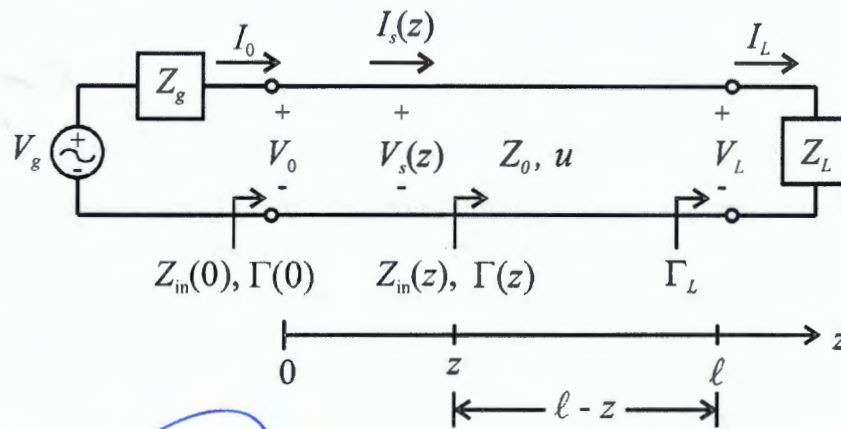
$$Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 75 \frac{1 + 0.4953 \angle 0.765^\circ}{1 - 0.4953 \angle 0.765^\circ} \\ = 222.11 + j3.89 \Omega$$

$$\beta = \text{phase constant} = 7.5398 \text{ rad/m}$$

$$\Gamma_{in} = \text{input refl. coeff.} = 0.4953 \angle 0.765^\circ$$

$$\Gamma_L = \text{load refl. coeff.} = 0.4953 \angle 130.365^\circ$$

$$Z_{in} = \text{input impedance} = 222.11 + j3.89 \Omega$$



$$u = f\lambda = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \text{ (m/s)}, \quad \beta = \frac{\omega}{u} = \omega\sqrt{LC} = \frac{2\pi}{\lambda} \text{ (rad/m)}, \quad \lambda = \frac{u}{f} = \frac{2\pi}{\beta} \text{ (m)}$$

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} = V_0^+ e^{-j\beta z} [1 + \Gamma(z)], \quad I_s(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z} = \frac{V_0^+}{Z_0} e^{-j\beta z} [1 - \Gamma(z)]$$

$$V_s(0) = V_0 = V_0^+ + V_0^-, \quad I_s(0) = I_0 = I_0^+ + I_0^- = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

$$V_0^+ = 0.5(V_0 + I_0 Z_0) = \frac{V_0}{1 + \Gamma(0)} = 0.5(V_L + I_L Z_0) e^{+j\beta \ell} \text{ and } I_0^+ = \frac{V_0^+}{Z_0}$$

$$V_0^- = 0.5(V_0 - I_0 Z_0) = V_0 - V_0^+ = 0.5(V_L - I_L Z_0) e^{-j\beta \ell} \text{ and } I_0^- = \frac{-V_0^-}{Z_0}$$

$$\Gamma(z) = \frac{V_{\text{ref}}(z)}{V_{\text{inc}}(z)} = -\frac{I_{\text{ref}}(z)}{I_{\text{inc}}(z)} = \frac{Z_{\text{in}}(z) - Z_0}{Z_{\text{in}}(z) + Z_0} = \Gamma_L e^{-j2\beta(\ell-z)} \text{ for } 0 \leq z \leq \ell,$$

$$\Gamma_L = \frac{V_0^-}{V_0^+} e^{+j2\beta \ell} = \Gamma(0) e^{+j2\beta \ell} = \frac{Z_L - Z_0}{Z_L + Z_0} \text{ and } \Gamma_{\text{in}} = \Gamma(0) = \frac{Z_{\text{in}}(0) - Z_0}{Z_{\text{in}}(0) + Z_0} = \Gamma_L e^{-j2\beta \ell}$$

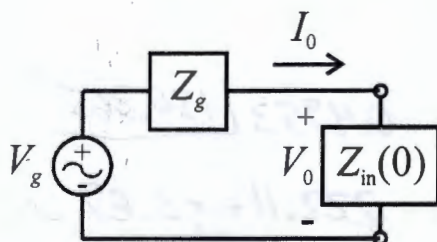
$$Z_{\text{in}}(z) = \frac{V(z)}{I(z)} = Z_0 \left[\frac{Z_L + jZ_0 \tan[\beta(\ell-z)]}{Z_0 + jZ_L \tan[\beta(\ell-z)]} \right] = Z_0 \left[\frac{1 + \Gamma(z)}{1 - \Gamma(z)} \right], \quad Z_L = Z_{\text{in}}(\ell) = \frac{V_L}{I_L} = Z_0 \left[\frac{1 + \Gamma_L}{1 - \Gamma_L} \right]$$

$$Z_{\text{in}} = Z_{\text{in}}(0) = \frac{V(0)}{I(0)} = \frac{V_0}{I_0} = Z_0 \left[\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \right] = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta \ell)}{Z_0 + jZ_L \tan(\beta \ell)} \right] = Z_0 \left[\frac{1 + \Gamma(0)}{1 - \Gamma(0)} \right] = Z_0 \left[\frac{1 + \Gamma_{\text{in}}}{1 - \Gamma_{\text{in}}} \right]$$

$$SWR = S = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{I_{\text{max}}}{I_{\text{min}}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$P_{\text{ave}}(z) = 0.5 \text{Re}\{V(z)I^*(z)\} = 0.5 \frac{|V_0^+|^2}{Z_0} [1 - |\Gamma(z)|^2] = P_{\text{ave}}(0) = 0.5 \text{Re}\{V_0 I_0^*\} = 0.5 \frac{|V_0^+|^2}{Z_0} [1 - |\Gamma(0)|^2]$$

$$= P_{\text{ave}}(\ell) = 0.5 \text{Re}\{V_L I_L^*\}$$

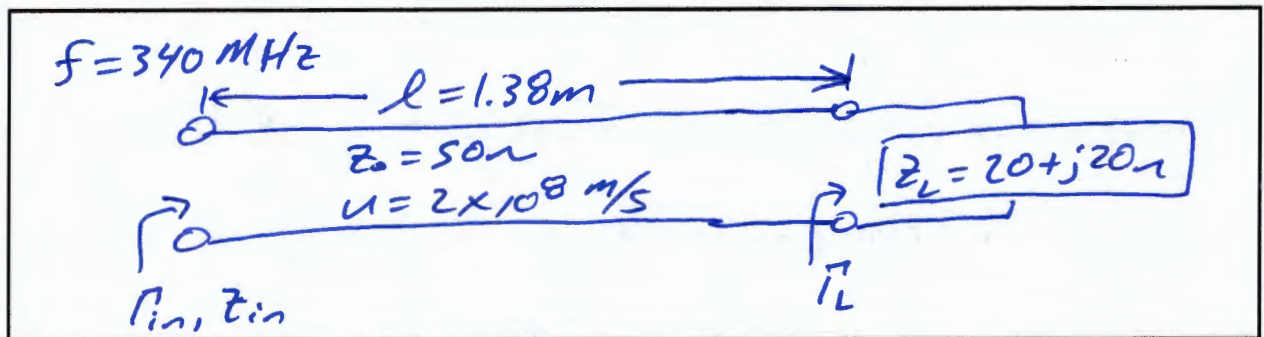


$$V_0 = V_g \left(\frac{Z_{\text{in}}(0)}{Z_g + Z_{\text{in}}(0)} \right) \text{ and } I_0 = \frac{V_g}{Z_g + Z_{\text{in}}(0)}$$

EE 381 Electric and Magnetic Fields Quiz #2 (Fall 2026)

Name Key BInstructions: **Closed** book & notes (see back). Place answers in indicated spaces and show all work for credit.

A lossless transmission line has a characteristic impedance of $50\ \Omega$ and phase velocity of $2 \times 10^8\ \text{m/s}$. If it is operated at $340\ \text{MHz}$, find the phase constant for this transmission line. If a $1.38\ \text{m}$ long section of the transmission line is terminated with a $20 + j20\ \Omega$ load, sketch a fully labeled drawing of this transmission line circuit in box. Then, find the load and input reflection coefficients and input impedance. [Note: Express currents, voltages, & reflection coefficients in polar format w/ angles in degrees. Express impedances in rectangular format.]



$$\beta = \omega/u = \frac{2\pi(340 \times 10^6)}{2 \times 10^8} = 10.6814 \text{ rad/m}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(20 + j20) - 50}{(20 + j20) + 50} = 0.4953 \angle 130.365^\circ$$

$$\Gamma_{in} = \Gamma_L e^{-j2\beta l} = (0.4953 \angle 130.365^\circ) e^{-j2(10.6814)(1.38)} \\ = 0.4953 \angle -118.76^\circ$$

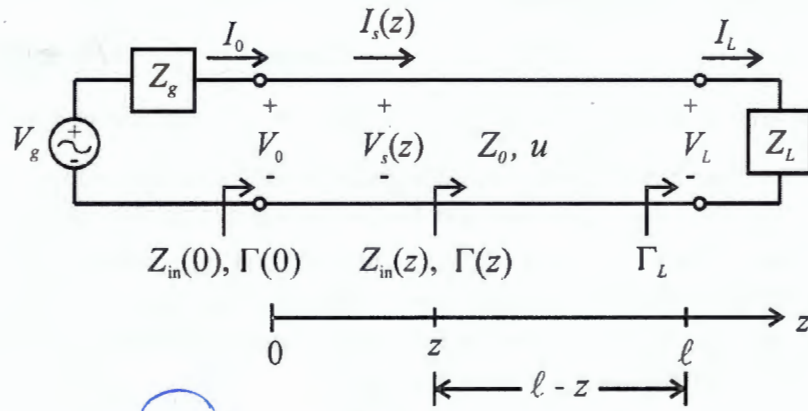
$$Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 50 \frac{1 + 0.4953 \angle -118.76^\circ}{1 - 0.4953 \angle -118.76^\circ} \\ = 21.92 - j25.22 \Omega$$

$$\beta = \text{phase constant} = 10.6814 \text{ rad/m}$$

$$\text{input refl. coeff.} = 0.4953 \angle -118.76^\circ$$

$$\Gamma_L \text{ load refl. coeff.} = 0.4953 \angle 130.365^\circ$$

$$\text{input impedance} = 21.92 - j25.22 \Omega$$



$$u = f\lambda = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \text{ (m/s)}, \quad \beta = \frac{\omega}{u} = \omega\sqrt{LC} = \frac{2\pi}{\lambda} \text{ (rad/m)}, \quad \lambda = \frac{u}{f} = \frac{2\pi}{\beta} \text{ (m)}$$

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} = V_0^+ e^{-j\beta z} [1 + \Gamma(z)], \quad I_s(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z} = \frac{V_0^+}{Z_0} e^{-j\beta z} [1 - \Gamma(z)]$$

$$V_s(0) = V_0 = V_0^+ + V_0^-, \quad I_s(0) = I_0 = I_0^+ + I_0^- = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

$$V_0^+ = 0.5(V_0 + I_0 Z_0) = \frac{V_0}{1 + \Gamma(0)} = 0.5(V_L + I_L Z_0) e^{+j\beta\ell} \text{ and } I_0^+ = \frac{V_0^+}{Z_0}$$

$$V_0^- = 0.5(V_0 - I_0 Z_0) = V_0 - V_0^+ = 0.5(V_L - I_L Z_0) e^{-j\beta\ell} \text{ and } I_0^- = \frac{-V_0^-}{Z_0}$$

$$\Gamma(z) = \frac{V_{\text{ref}}(z)}{V_{\text{inc}}(z)} = -\frac{I_{\text{ref}}(z)}{I_{\text{inc}}(z)} = \frac{Z_{\text{in}}(z) - Z_0}{Z_{\text{in}}(z) + Z_0} = \Gamma_L e^{-j2\beta(\ell-z)} \text{ for } 0 \leq z \leq \ell,$$

$$\Gamma_L = \frac{V_0^-}{V_0^+} e^{+j2\beta\ell} = \Gamma(0) e^{+j2\beta\ell} = \frac{Z_L - Z_0}{Z_L + Z_0} \text{ and } \Gamma_{\text{in}} = \Gamma(0) = \frac{Z_{\text{in}}(0) - Z_0}{Z_{\text{in}}(0) + Z_0} = \Gamma_L e^{-j2\beta\ell}$$

$$Z_{\text{in}}(z) = \frac{V(z)}{I(z)} = Z_0 \left[\frac{Z_L + jZ_0 \tan[\beta(\ell-z)]}{Z_0 + jZ_L \tan[\beta(\ell-z)]} \right] = Z_0 \left[\frac{1 + \Gamma(z)}{1 - \Gamma(z)} \right], \quad Z_L = Z_{\text{in}}(\ell) = \frac{V_L}{I_L} = Z_0 \left[\frac{1 + \Gamma_L}{1 - \Gamma_L} \right]$$

$$Z_{\text{in}} = Z_{\text{in}}(0) = \frac{V(0)}{I(0)} = \frac{V_0}{I_0} = Z_0 \left[\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \right] = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)} \right] = Z_0 \left[\frac{1 + \Gamma(0)}{1 - \Gamma(0)} \right] = Z_0 \left[\frac{1 + \Gamma_{\text{in}}}{1 - \Gamma_{\text{in}}} \right]$$

$$SWR = S = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{I_{\text{max}}}{I_{\text{min}}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$P_{\text{ave}}(z) = 0.5 \text{Re}\{V(z)I^*(z)\} = 0.5 \frac{|V_0^+|^2}{Z_0} [1 - |\Gamma(z)|^2] = P_{\text{ave}}(0) = 0.5 \text{Re}\{V_0 I_0^*\} = 0.5 \frac{|V_0^+|^2}{Z_0} [1 - |\Gamma(0)|^2]$$

$$= P_{\text{ave}}(\ell) = 0.5 \text{Re}\{V_L I_L^*\}$$

