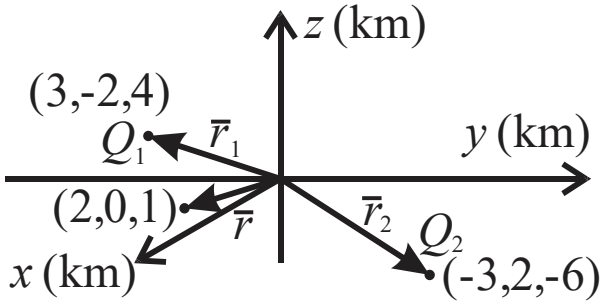
EE 381 Electric & Magnetic Fields Examination #2 (Fall 2026)

Name gruesome edition

Instructions: Place answers in indicated spaces. Use notation as given in class for coordinates and vectors. Use standard units. **Show all work** for full credit. Insert equation sheets inside exam.

1) Two Sunstone crystals, one imbued with 10 μC and located at (3, -2, 4) while the other is charged with -15 μC and located at (-3, 2, -6) are placed in free space near the Fortress of Solitude. Calculate the absolute electrostatic potential V_C due to these crystals at (2, 0, 1). All spatial coordinates are in units of km.



Per (4.66), $V = \sum_{k=1}^{n} \frac{Q_k}{4\pi\varepsilon_0 |\bar{r} - \bar{r}_k|} = \frac{Q_1}{4\pi\varepsilon_0 |\bar{r} - \bar{r}_1|} + \frac{Q_2}{4\pi\varepsilon_0 |\bar{r} - \bar{r}_2|}$ for multiple point charges.

The field position vector is $\bar{r} = 2\hat{a}_x + \hat{a}_z$ (km).

Let $Q_1 = 10\ \mu$C & $\bar{r}_1 = 3\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z$ (km) and $Q_2 = $ -15 μC & $\bar{r}_2 = -3\hat{a}_x + 2\hat{a}_y - 6\hat{a}_z$ (km) for the point charges.

Then, the distances are:

$$|\bar{r} - \bar{r}_1| = \left|(2000 - 3000)\hat{a}_x + 2000\hat{a}_y + (1000 - 4000)\hat{a}_z\right| = \left|-1000\hat{a}_x + 2000\hat{a}_y - 3000\hat{a}_z\right|$$
$$= \sqrt{(-1000)^2 + 2000^2 + (-3000)^2} = 3741.657387 \text{ m}$$

$$|\bar{r} - \bar{r}_2| = \left|(2000 + 3000)\hat{a}_x - 2000\hat{a}_y + (1000 + 6000)\hat{a}_z\right| = \left|5000\hat{a}_x - 2000\hat{a}_y + 7000\hat{a}_z\right|$$
$$= \sqrt{5000^2 + (-2000)^2 + 7000^2} = 8831.760866 \text{ m}$$

Plugging these into (4.66),

$$V_S = \frac{10\times10^{-6}}{4\pi(8.8541878\times10^{-12})3741.66} + \frac{-15\times10^{-6}}{4\pi(8.8541878\times10^{-12})8831.76}$$
$$= 24.02024 - 15.2646$$
$$= 8.755645 \text{ V}$$

$V_C =$ **8.7556 V**

2) A gruesome vector field $\bar{G} = \frac{4}{\rho}\hat{a}_\rho - 8\rho^2 z\,\hat{a}_\phi + 3z\cos\phi\,\hat{a}_z$ (ghouls/m^2) exists at a graveyard, find the divergence and curl of $\bar{G}$. Then, find the outward flux Ψ_G of $\bar{G}$ through the closed surface described by $2 \le \rho \le 4$ m, $0 < \phi < \pi/4$, and $0 \le z \le 2$ m.

$$\text{div } \bar{G} = \bar{\nabla}\bullet\bar{G} = \frac{1}{\rho}\frac{\partial(\rho G_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial G_\phi}{\partial\phi} + \frac{\partial G_z}{\partial z} = \frac{1}{\rho}\frac{\partial(4)}{\partial\rho} + \frac{1}{\rho}\frac{\partial(-8\rho^2 z)}{\partial\phi} + \frac{\partial(3z\cos\phi)}{\partial z}$$

$$= 0 + 0 + 3\cos\phi = 3\cos\phi \text{ (ghouls/m}^3\text{)}$$

$$\text{curl } \bar{G} = \bar{\nabla}\times\bar{G} = \hat{a}_\rho\left(\frac{1}{\rho}\frac{\partial G_z}{\partial\phi} - \frac{\partial G_\phi}{\partial z}\right) + \hat{a}_\phi\left(\frac{\partial G_\rho}{\partial z} - \frac{\partial G_z}{\partial\rho}\right) + \hat{a}_z\left(\frac{1}{\rho}\frac{\partial(\rho G_\phi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial G_\rho}{\partial\phi}\right)$$

$$= \hat{a}_\rho\left(\frac{1}{\rho}\frac{\partial(3z\cos\phi)}{\partial\phi} - \frac{\partial(-8\rho^2 z)}{\partial z}\right) + \hat{a}_\phi\left(\frac{\partial(4/\rho)}{\partial z} - \frac{\partial(3z\cos\phi)}{\partial\rho}\right)$$

$$+ \hat{a}_z\left(\frac{1}{\rho}\frac{\partial(-8\rho^3 z)}{\partial\rho} - \frac{1}{\rho}\frac{\partial(4/\rho)}{\partial\phi}\right)$$

$$= \hat{a}_\rho\left(\frac{-3z\sin\phi}{\rho} + 8\rho^2\right) + \hat{a}_\phi(0-0) + \hat{a}_z\left(\frac{-24\rho^2 z}{\rho} - 0\right) \text{ (ghouls/m}^3\text{)}$$

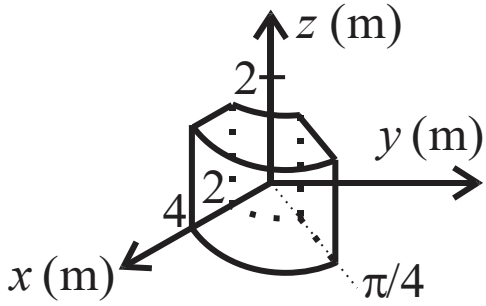
Use divergence theorem to get the outward flux Ψ_G of $\bar{G}$ through the closed surface described by $2 \le \rho \le 4$ m, $0 < \phi < \pi/4$, and $0 \le z \le 2$ m.

$$\Psi_G = \oiint_S \bar{G}\bullet d\bar{s} = \iiint_V \bar{\nabla}\bullet\bar{G}\,dv = \iiint_V 3\cos\phi\;\rho\,d\rho\,d\phi\,dz$$

$$= 3\int_{z=0}^{2} dz \int_{\phi=0}^{\pi/4}\cos\phi\,d\phi \int_{\rho=2}^{4}\rho\,d\rho$$

$$= 3(2-0)(\sin\phi)\Big|_{\phi=0}^{\pi/4}\left(\frac{\rho^2}{2}\right)\Big|_{\rho=2}^{4}$$

$$= 6(\sin\pi/4 - 0)(16/2 - 4/2) = 25.4558 \text{ (ghouls)}$$



curl $\bar{G} = \bar{\nabla}\times\bar{G} = \left(\frac{-3z\sin\phi}{\rho} + 8\rho^2\right)\hat{a}_\rho - 24\rho z\,\hat{a}_z$ **(ghouls /m^3)**

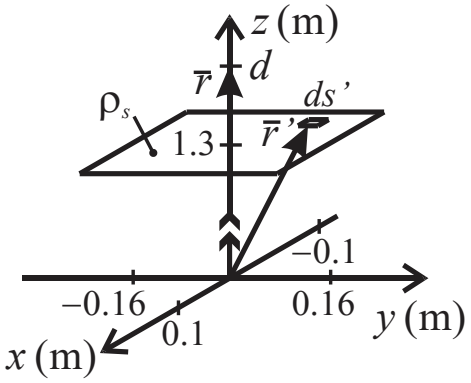
div $\bar{G}$ = **3 cos ϕ (ghouls/m^3)** Ψ_G = **25.456 (ghouls)**

Extra Credit: Is $\bar{G}$: divergenceless? Y / **N**; irrotational? Y / **N**; conservative? Y / **N**

3) To create Elixir of Life (AKA: Elixir Vitae), Johann Konrad Dippel has placed a uniform surface charge density of -48 μC/m^2 on a laboratory bench, a rectangular surface ($-10 \le x \le 10$ cm, $-16 \le y \le 16$ cm, & $z = 130$ cm) suspended in free space. Find indicated quantities needed to calculate the electric field at a point (0, 0, d) along the z-axis above the bench. (Do **NOT** attempt to solve, simplify, or perform any coordinate transformations.)

a) Circle most appropriate coordinate system for problem- Cartesian

b) Draw a fully-labeled sketch of the problem geometry, include differentials, position vectors ...



c) Appropriate general integral to find $\bar{E}$ in position vector form (do NOT define terms or set limits) is:

$$\bar{E} = \iint_{s'} \frac{\rho_s (\bar{r} - \bar{r}')\, ds'}{4\pi\varepsilon_0 |\bar{r} - \bar{r}'|^3}$$

d) Determine specific **field** position vector:

At point (0, 0, d), $\bar{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z = d\,\hat{a}_z$

$$\bar{r} = d\,\hat{a}_z$$

e) Determine specific **source** position vector:

On surface for ρ_s, $\bar{r}' = x'\,\hat{a}_x + y'\,\hat{a}_y + z'\,\hat{a}_z = x'\,\hat{a}_x + y'\,\hat{a}_y + 1.3\,\hat{a}_z$ (m)

$$\bar{r}' = x'\,\hat{a}_x + y'\,\hat{a}_y + 1.3\,\hat{a}_z \text{ (m)}$$

f) Determine **source** differential, i.e., circle correct one, and write-out specific expression

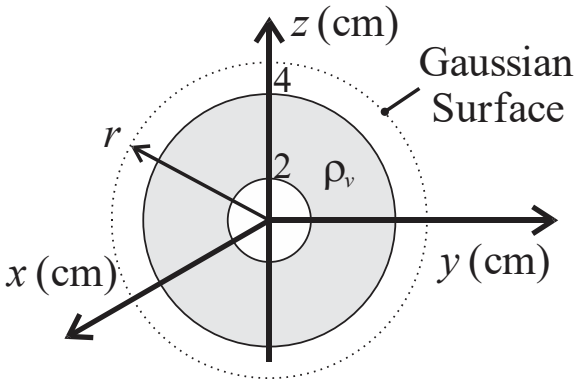
$$ds' = |d\bar{s}_z'| = dx'dy'$$

g) Use above to set-up the specific integral for $\bar{E}$, including limits. Do **NOT** simplify or solve.

$$\bar{E} = \int_{y'=-0.16}^{0.16} \int_{x'=-0.1}^{0.1} \frac{-48\times10^{-6}\,[d\,\hat{a}_z - (x'\,\hat{a}_x + y'\,\hat{a}_y + 1.3\,\hat{a}_z)]\,dx'dy'}{4\pi(8.854\times10^{-12})\left|d\,\hat{a}_z - (x'\,\hat{a}_x + y'\,\hat{a}_y + 1.3\,\hat{a}_z)\right|^3} \text{ (V/m)}$$

4) To control her flying monkeys, the Wicked Witch of the West creates a ball of witchfire, i.e., a uniform volume charge density $\rho_v = 6\ \mu C/m^3$ bound in a spherical shell $2 \le r \le 4$ cm hovering in free space at the origin. Find the total charge Q_{WWW} contained in the plasma ball. Then, find the electric flux density vectors at spherical points F(1.8 cm, π/2, 3π/2) & M(5.8 cm, π/3, 4π/3). (Hint: Gauss' Law)

$$Q_{WWW} = \iiint_V \rho_v\, dV = \iiint_V 6\times10^{-6}\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$
$$= 6\times10^{-6}\int_{r=0.02}^{0.04} r^2\, dr \int_{\theta=0}^{\pi} \sin\theta\, d\theta \int_{\phi=0}^{2\pi} d\phi$$
$$= 6\times10^{-6}\left(\frac{r^3}{3}\right)\Bigg|_{r=0.02}^{0.04} (-\cos\theta)\Big|_{\theta=0}^{\pi} (\phi)\Big|_{\phi=0}^{2\pi}$$
$$= 6\times10^{-6}(0.04^3/3 - 0.02^3/3)(1+1)(2\pi - 0)$$
$$= 1.4074335\times10^{-9}\ \text{(C)}$$



Per Gauss' Law $\psi = \oiint_s \bar{D}\bullet d\bar{s} = Q_{\text{enc}}$. By spherical symmetry, we expect the electric flux to be in the radial direction, $\bar{D} = \hat{a}_r D_r$ and constant at a given radius r. Choose a sphere of radius r as our Gaussian surface. Then,

$$\oiint_s \bar{D}\bullet d\bar{s} = \oiint_s \hat{a}_r D_r \bullet d\bar{s}_r = D_r \oiint_s ds_r = D_r 4\pi r^2.$$

At point F(1.8 cm, π/2, 3π/2), we are **inside** the plasma ball. Therefore, $Q_{\text{enc}} = 0$ and we get

$$\oiint_s \bar{D}\bullet d\bar{s} = D_{r,F} 4\pi(0.018)^2 = Q_{\text{enc},F} = 0 \Rightarrow \underline{\boldsymbol{D_{r,F} = 0}}.$$

At point M(5.8 cm, π/3, 4π/3), we are **outside** the plasma ball. Therefore, $Q_{\text{enc}} = Q_{WWW}$ and we get

$$\oiint_s \bar{D}\bullet d\bar{s} = D_{r,M} 4\pi(0.058)^2 = Q_{\text{enc},M} = 1.4074335\times10^{-9} \Rightarrow \underline{\boldsymbol{D_{r,M} = 3.3293698\times10^{-8}\ (C/m^2)}}.$$

$Q_{WWW} =$ **1.40743 (nC)** $\bar{D}_F =$ **0** $\bar{D}_M =$ $\hat{a}_r$ 33.2937 **(nC/m²)**